Coefficient of performance for a low-dissipation Carnot-like refrigerator with nonadiabatic dissipation

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We study the coefficient of performance (COP) and its bounds for a Carnot-like refrigerator working between two heat reservoirs at constant temperatures T_h and T_c, under two optimization criteria χ and Ω. In view of the fact that an "adiabatic" process usually takes finite time and is nonisentropic, the nonadiabatic dissipation and the finite time required for the adiabatic processes are taken into account by assuming low dissipation. For given optimization criteria, we find that the lower and upper bounds of the COP are the same as the corresponding ones obtained from the previous idealized models where any adiabatic process is undergone instantaneously with constant entropy. To describe some particular models with very fast adiabatic transitions, we also consider the influence of the nonadiabatic dissipation on the bounds of the COP, under the assumption that the irreversible entropy production in the adiabatic process is constant and independent of time. Our theoretical predictions match the observed COPs of real refrigerators more closely than the ones derived in the previous models, providing a strong argument in favor of our approach.

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I. INTRODUCTION

The issue of thermodynamic optimization of cyclic converters has attracted much attention because of sustainable development in relation to any energy converter operation. Concerning this issue, a number of different performance regimes [1–3] have been considered within different figures of merit to disclose possible universal and unified features, with special emphasis on the possible consistency between theoretical predictions and experimental data. If heat engines, or refrigerators, work between two heat reservoirs at constant temperatures T_h and T_c, in practice they operate far from the ideal maximum Carnot efficiency ($\eta_{\max} = \eta_C = 1 - T_c/T_h$), or the maximum Carnot coefficient of performance (COP) [$\varepsilon_{\max} = \varepsilon_C = T_c/(T_h - T_c)$], which requires infinite time to complete a cycle. By contrast, the maximum output for heat engines, or the maximum cooling rate for refrigerators, can be achieved within a finite cycle time. In most studies of Carnot-like heat-engine models, the power output as a target function is always maximized to find valuable and simple expressions for the optimized efficiency [4–13]. Without assuming any specific heat transfer law or the linear-response regime, Esposito *et al*. [11] proposed the low-dissipation assumption that the irreversible entropy production in a heat-exchange process is inversely proportional to the time spent on the corresponding process, and they rederived the paradigmatic Curzon-Ahlborn value [14] $\eta_{CA} = 1 - \sqrt{1 - \eta_C}$ in the limit of symmetric dissipation. In addition to the power output, the per-unit-time efficiency, a compromise between the efficiency and the speed of the whole heat-engine cycle, was considered as another criterion [15] of optimization.

It is more difficult to adopt a suitable optimization criterion and determine its corresponding COP for refrigerators, in comparison with dealing with the issue of the efficiency at maximum power for heat engines. Various optimization criteria [16–22] have been proposed in analysis of optimization of a classical or quantum refrigeration cycle. Yan and Chen [16] introduced the function $\chi = \varepsilon Q_c/\tau_{\text{cycle}}$, with Q_c the heat transported from the cold reservoir and τ_{cycle} the cycle time, as a target function within the context of finite-time thermodynamics. Velasco *et al*. [17] adopted the per-unit-time COP as a target function while Allahverdyan *et al*. [18] introduced εQ_c as the target function. de Tomás *et al*. [20] proved that the COP at maximum χ for symmetric low-dissipation refrigerators is $\varepsilon_{CA} = \sqrt{\varepsilon_C + 1} - 1$, where $\varepsilon_C = T_c/(T_h - T_c)$ is the Carnot COP. Based on the χ figure of merit, Wang *et al*. [21] obtained the lower and upper bounds of the COP and showed that these bounds can be achieved in extremely asymmetric dissipation limits. Very recently, de Tomás *et al*. [19] studied low-dissipation heat devices and obtained the bounds of the COP under general and symmetric conditions, by applying the unified Ω optimization criterion, which was first proposed in [23] to consider a compromise between energy benefits and losses for a specific job. This criterion, which takes advantage of being independent of environmental parameters and does not require explicit evaluation of the entropy generation, has been applied to performance optimization for a wide variety of energy converters [24–26].

Most of the previous studies about the performance in finite time of heat devices did not take into account nonadiabatic dissipation for the cyclic converter, assuming that the adiabatic steps run instantaneously with constant entropy, although the importance of nonadiabatic dissipation in an adiabatic process was suggested by Novikov [27]. The influence on the performance of a classical or quantum heat engine, induced by internally dissipative dissipation (such as inner friction and internal dynamics, etc.), has been discussed in several papers [28–35]. To the best of our knowledge, so far little attention has been paid to the effects of nonadiabatic dissipation on the

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performance characteristics of refrigerators. It is therefore of significance to consider a more generalized refrigerator model by involving the nonadiabatic dissipation and the time spent on adiabatic processes.

In the present paper, we consider a low-dissipation Carnot-like refrigeration cycle of two irreversible isothermal and two irreversible adiabatic processes, and analyze its COP under the χ and Ω conditions, respectively. Assuming that the nonadiabatic dissipation can be described using the low-dissipation assumption, we show that the inclusion of nonadiabatic dissipation does not lead to any change in the bounds of the COP at a given figure of merit. When the dissipations of the two isothermal and two adiabatic processes are symmetric, we find that our results agree well with the data for real refrigerators, thereby indicating that inclusion of nonadiabatic dissipation is essential. Moreover, we briefly discuss the influence of the nonadiabatic dissipation on the bounds of the COP with the χ and Ω figures of merit, by assuming that the entropy production in the adiabatic process is constant and independent of time. Throughout the paper, we use the word "isothermal" to mean that the working substance is coupled to a reservoir with constant temperature, while we adopt the word "adiabatic" to indicate merely that there are no heat exchanges between the working substance and its surroundings.

II. MODEL

An irreversible Carnot-like refrigeration cycle $A \to B \to C \to D \to A$ is drawn in the (S,T) plane (see Fig. 1). During two isothermal processes $A \to B$ and $C \to D$, the working substance is in contact with a cold and a hot heat bath at constant temperatures T_c and T_h, respectively. In the adiabatic process $B \to C$ ($D \to A$), the working substance is decoupled from the cold (hot) reservoir, and the entropy changes from S_B to S_C (S_D to S_A). It can be seen from Fig. 1 that $S_1 = S_A$, $S_2 = S_B$, $S_3 = S_C$, and $S_4 = S_D$. For the reversible cycle where $S_B = S_C$ and $S_A = S_D$, we recover the Carnot efficient of performance $\varepsilon_C = \frac{T_c}{T_h - T_c}$, which is generically universal.

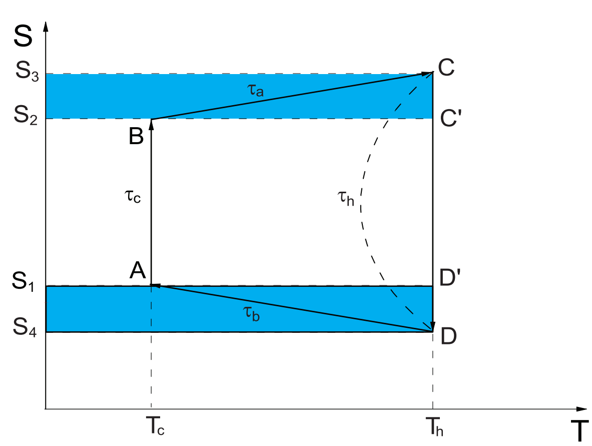

FIG. 1. (Color online) Schematic diagram of an irreversible Carnot-like refrigeration cycle in the plane of the temperature T and entropy S. The values of the entropy S at four special instants are indicated by S_i ($i = 1,3,3,4$). Here $\tau_{h,c}$ are the times spent on the two isothermal process, while $\tau_{a,b}$ represent the times taken for the two adiabatic processes. The blue areas defined by the rectangles C,C',S_2,S_3 and D,D',S_1,S_4 represent the total work done on the system to overcome nonadiabatic dissipation in the two adiabatic processes.

Now we turn to discussion of the Carnot-like cycle under a finite-time operation that moves the working substance away from equilibrium. In the isothermal process the system may be out of equilibrium, but it must be in the equilibrium with the heat reservoir at the special instants i with $i = A,B,C,D$, at which the thermodynamic quantities of the system can be defined well. Unlike in the ideal case where any adiabatic process is isentropic, the adiabatic process is nonisentropic because of nonadiabatic dissipation. This dissipation develops additional heat and thus yields an increase in the entropy during the so-called adiabatic process. An irreversible Carnot-like refrigerator that consists of two adiabatic and two isothermal processes is operated as follows (more details about the isothermal processes can be seen in [21]).

(1) Isothermal expansion $A \to B$. The working substance is in contact with the cold reservoir at temperature T_c for a period τ_c. In this expansion the constraint imposed on the system is loosened according to the externally controlled parameter $\lambda_c(\tau)$ during the time interval $0 < \tau < \tau_c$, where τ is the time variable. A certain amount of heat Q_c is absorbed from the cold reservoir and the variation of entropy can be expressed as

$$\Delta S_c = Q_c/T_c + \Delta S_c^{ir}, \tag{1}$$

with $\Delta S_c^{ir} \geqslant 0$ being the irreversible entropy production.

(2) Adiabatic compression $B \to C$. The entropy is increased due to irreversible entropy production caused by the nonadiabatic dissipation, while the constraint on the system is enhanced according to the externally controlled parameter $\lambda_a(\tau)$ during the time interval $\tau_c < \tau < \tau_c + \tau_a$. The irreversible entropy production arising from the nonadiabatic dissipation is denoted by

$$\Delta S_a^{ir} = S_3 - S_2. \tag{2}$$

(3) Isothermal compression $C \to D$. The working substance is coupled to a hot reservoir at constant temperature T_h for time τ_h. The constraint on the system is further enhanced with the externally controlled parameter $\lambda_h(\tau)$ during the time interval $\tau_c + \tau_a < \tau < \tau_c + \tau_a + \tau_h$. Let Q_h be the amount of heat released to the hot reservoir; we have the entropy variation

$$\Delta S_h = -Q_h/T_h + \Delta S_h^{ir}, \tag{3}$$

where $\Delta S_h^{ir} \geqslant 0$ is the irreversible entropy production.

(4) Adiabatic expansion $D \to A$. As in adiabatic compression, the working substance is decoupled from the hot reservoir. During this process, the controlled parameter $\lambda_b(\tau)$ changes from $\lambda_b(\tau_c + \tau_a + \tau_h)$ to $\lambda_b(\tau_c + \tau_a + \tau_h + \tau_b)$, so the constraint on the system is loosened. The entropy production due to the nonadiabatic dissipation reads

$$\Delta S_b^{ir} = S_1 - S_4. \tag{4}$$

The system recovers to its initial state after a single cycle, and the total change of entropy of the system vanishes for a whole cycle. That is, there exists the following relation:

$$\Delta S + \Delta S_a^{ir} + \Delta S_h + \Delta S_b^{ir} = 0, \tag{5}$$

where we have defined $\Delta S \equiv \Delta S_c = S_2 - S_1$. It is clear from Eq. (5) that the variation of entropy ΔS ($=\Delta S_c$) during the cold isothermal process is quite different from that ΔS_h during the hot isothermal process, when the entropy production during any adiabatic process is nonvanishing.

To continue our analysis, we denote by $\Delta S_\kappa^{ir} \equiv L_\kappa(t_\kappa)$ [11,21,29,34] with $\kappa = h,c,a,b$ the irreversible entropy production for the optimized protocols. As emphasized, the irreversible entropy production in any adiabatic process [$L_a(\tau_a)$ or $L_b(\tau_b)$] cannot be included within the irreversible entropy production in any isothermal process [$L_c(\tau_c)$ or $L_h(\tau_h)$], because the irreversible entropy production $L_\kappa(\tau_\kappa)$ as a function of the time τ_κ depends on the time taken for the corresponding process κ. Here $L_\kappa(\tau_\kappa)$ are process variables depending on the detailed protocols.

Considering Eqs. (1), (2), (3), (4), and (5), the heat values Q_c and Q_h are obtained,

$$Q_c = T_c(\Delta S - L_c) \tag{6}$$

and

$$Q_h = T_h(\Delta S + L_a + L_b + L_h). \tag{7}$$

From Eqs. (6) and (7), we find the relation

$$Q_h/T_h - Q_c/T_c = L_a + L_b + L_h + L_c. \tag{8}$$

As a consequence, the work consumed by the system after a single cycle (W) and the COP of the refrigeration cycle (ε) are derived as

$$\begin{aligned} W = Q_h - Q_c = (T_h - T_c)\Delta S + T_h L_h + T_c L_c \\ + T_h(L_a + L_b) \end{aligned} \tag{9}$$

and

$$\begin{aligned} \varepsilon &= \frac{Q_c}{Q_h - Q_c} \\ &= \frac{T_c(\Delta S - L_c)}{(T_h - T_c)\Delta S + T_h(L_a + L_b + L_h) + T_c L_c}. \end{aligned} \tag{10}$$

The last term in Eq. (9) represents the additional work consumed by the system because of the dissipation in the two adiabatic processes. This additional work to overcome the internally nonadiabatic dissipation is represented by the two blue areas in Fig. 1.

III. OPTIMIZATION ANALYSIS

In this section we present an optimization analysis of a refrigerator with internal dissipation which accounts for the irreversible entropy production during a nonisentropic adiabatic process (more details about nonisentropic adiabatic processes can be found in Ref. [29]). If the adiabatic processes are assumed to proceed instantaneously with constant entropy, we recall that [19,21] (i) the bounds of the COP under the Ω criterion, between which there are small differences, are in agreement with the real experimental data within a range of temperatures of the working substance; (ii) under the χ criterion, the upper bound of the COP fits well with the experimental data, but the COP in the symmetric limit ($\varepsilon_\chi^{\Sigma_h=\Sigma_c}$) seems to be considerably larger than the experimental data. In what follows, our theoretical predictions are expected to agree well with the experimental data. In particular, for the χ criterion, our theoretical data in the symmetric limit should match more closely with the experimental data than the ones obtained from the previous models without consideration of nonadiabatic dissipation [19].

A. COP at maximum χ figure of merit

In the following, we make the variable transformation $x_\kappa = 1/\tau_\kappa$ ($\kappa = a,b,c,h$) by taking the inverse of time instead of the time itself as a variable, and then we can write the total cycle time τ_{cycle} as $\tau_{\text{cycle}} = 1/x_a + 1/x_b + 1/x_c + 1/x_h$. Substitution of Eqs. (6) and (10) into the χ figure of merit as the target function leads to

$$\chi = \frac{\varepsilon Q_c}{\tau_{\text{cycle}}} = \frac{T_c^2(\Delta S - L_c)^2}{[(T_h - T_c)\Delta S + T_h(L_a + L_b + L_h) + T_c L_c](1/x_a + 1/x_b + 1/x_c + 1/x_h)}. \tag{11}$$

We optimize the target function χ over the time variables x_κ to specify the time spent on any thermodynamic process and also to maximize this figure of merit. Considering $\frac{\partial \chi}{\partial x_\kappa} = 0$ ($\kappa = a,b,c,h$), we find the four following relations:

$$\begin{aligned} (Q_h - Q_c)x_a x_b x_h = T_c L_c' x_c (2Q_h/Q_c - 1)(x_a x_b x_c + x_a x_b x_h \\ + x_b x_c x_h + x_a x_c x_h), \end{aligned} \tag{12}$$

$$\begin{aligned} (Q_h - Q_c)x_b x_c x_h = T_h L_a' x_a (x_a x_b x_c + x_a x_b x_h \\ + x_b x_c x_h + x_a x_c x_h), \end{aligned} \tag{13}$$

$$\begin{aligned} (Q_h - Q_c)x_a x_c x_h = T_h L_b' x_b (x_a x_b x_c + x_a x_b x_h \\ + x_b x_c x_h + x_a x_c x_h), \end{aligned} \tag{14}$$

$$\begin{aligned} (Q_h - Q_c)x_a x_b x_c = T_h L_h' x_h (x_a x_b x_c + x_a x_b x_h \\ + x_b x_c x_h + x_a x_c x_h). \end{aligned} \tag{15}$$

Here and hereafter we define $L_\kappa' \equiv \frac{\partial L_\kappa}{\partial x_\kappa}$ ($\kappa = a,b,c,h$). Dividing Eq. (12) by Eq. (13), Eq. (14), and Eq. (15), respectively, we obtain

$$\varepsilon_\chi^* T_h L_a' x_a^2 = (\varepsilon_\chi^* + 2) T_c L_c' x_c^2, \tag{16}$$

$$\varepsilon_\chi^* T_h L_b' x_b^2 = (\varepsilon_\chi^* + 2) T_c L_c' x_c^2, \tag{17}$$

$$\varepsilon_\chi^* T_h L_h' x_h^2 = (\varepsilon_\chi^* + 2) T_c L_c' x_c^2, \tag{18}$$

where ε_χ^* is the COP under the maximum χ condition. From Eqs. (16), (17), and (18), we find that the times spent on the four thermodynamic processes are optimally distributed as

$$\tau_\kappa/\tau_h = \sqrt{L'_\kappa/L'_h}(\kappa = a,b), \quad \tau_b/\tau_a = \sqrt{L'_b/L'_a}, \tag{19}$$

and

$$\tau_\kappa/\tau_c = \sqrt{T_h L'_\kappa/(m T_c L'_c)} \ (\kappa = a,b,h), \tag{20}$$

where $m \equiv (\varepsilon_\chi^* + 2)/\varepsilon_\chi^*$ has been adopted and can be determined through numerical calculation of ε_χ^*. Directly adding both sides of Eqs. (12), (13), (14), and (15), we obtain

$$Q_h - Q_c = (2Q_h/Q_c - 1)T_c L'_c x_c + T_h(L'_a x_a + L'_b x_b + L'_h x_h). \tag{21}$$

Based on Eqs. (8) and Eq. (21), it is not very difficult to find that (see the Appendix)

$$\frac{1}{\varepsilon_\chi^*} = \frac{1}{\varepsilon_C} + \frac{1}{\alpha_1\varepsilon_\chi^* + [(2\varepsilon_C - \varepsilon_\chi^*)\alpha_2/(1+\varepsilon_C)]} \tag{22}$$

or

$$\varepsilon_\chi^* = \frac{\varepsilon_C\{\alpha_1 - 1 - 3\alpha_2 + (\alpha_1 - 1)\varepsilon_C + \sqrt{[\alpha_1 - 1 - 3\alpha_2 + (\alpha_1-1)\varepsilon_C]^2 + 8\alpha_2(\alpha_1 - \alpha_2 + \alpha_1\varepsilon_C)}\}}{2(\alpha_1 - \alpha_2 + \alpha_1\varepsilon_C)}, \tag{23}$$

where we have used $\alpha_1 \equiv \frac{L'_a x_a + L'_b x_b + L'_c x_c + L'_h x_h}{L_a + L_b + L_c + L_h}$ and $\alpha_2 \equiv \frac{L'_c x_c}{L_a + L_b + L_c + L_h}$.

Now we turn to the low-dissipation case [11] where one assumes $L'_c = \Sigma_c$ and $L'_h = \Sigma_h$ with Σ_c and Σ_h being the dissipation constants. This assumption is quite plausible for isothermal processes, since the larger the time for completing the isothermal processes, the closer these steps are to quasistatic processes taking infinitely long times. Usually, for a Carnot (or Carnot-like) cycle that works with a classical gas, the time required for completing an adiabatic process should be very long in order for work to be produced during the adiabatic process (for a quantum adiabatic process, the time must be long enough such that the quantum adiabatic theorem can apply [29]). It is therefore indicated that the irreversible entropy production decreases as the time for the process increases, and it tends to be vanishing when the time becomes long enough. As for isothermal processes, we adopt the low-dissipation assumption for such an adiabatic process [28,29,32–34] to describe the nonadiabatic dissipation, assuming $L'_a = \Sigma_a$ and $L'_b = \Sigma_b$ where Σ_a and Σ_b are constants independent of time. This is physically reasonable since the irreversible entropy production (ΔS_a^{ir} or ΔS_b^{ir}) becomes much smaller and is vanishing in the long-time limit. In this case, since $\alpha_1 = 1$, Eq. (23) becomes

$$\varepsilon_\chi^* = \frac{\varepsilon_C[\sqrt{1 + 8(1+\varepsilon_C)/\alpha_2} - 3]}{2[(1+\varepsilon_C)/\alpha_2 - 1]}, \tag{24}$$

with

$$\alpha_2 = \frac{\Sigma_c x_c}{\Sigma_a x_a + \Sigma_b x_b + \Sigma_c x_c + \Sigma_h x_h}. \tag{25}$$

The expression for α_2 is derived from the more general model in which the nonadiabatic dissipation and the time spent on any adiabatic process are involved. Since $0 \leqslant \alpha_2 \leqslant 1$ and $\varepsilon_C > 0$, ε_χ^* increases monotonically with α_2, and vice versa. As a result, we rederive the bounds of the COP at the maximum χ figure of merit [21,36],

$$0 \equiv \varepsilon_\chi^- \leqslant \varepsilon_\chi^* \leqslant \varepsilon_\chi^+ \equiv (\sqrt{9 + 8\varepsilon_C} - 3)/2, \tag{26}$$

whether or not the nonadiabatic dissipation in any adiabatic process is considered. It is thus clear that the inclusion of the nonadiabatic dissipation as well as the time taken for the adiabatic process does not change the upper and lower bounds of the COP at the maximum χ figure of merit. These lower and upper bounds of ε_χ^* are achieved when $\alpha_2 \to 0$ and $\alpha_2 \to 1$, respectively. Combination of Eqs. (20) and (25) yields

$$\alpha_2 = \frac{1}{\sqrt{m}\left(\sqrt{\frac{T_c\Sigma_h}{T_h\Sigma_c}} + \sqrt{\frac{T_c\Sigma_a}{T_h\Sigma_c}} + \sqrt{\frac{T_c\Sigma_b}{T_h\Sigma_c}}\right) + 1}, \tag{27}$$

where m was defined in Eq. (20) and $x_\kappa = 1/\tau_\kappa$ ($\kappa = a,b,c,h$) has been used. The complete asymmetric limits $\Sigma_c/\Sigma_\kappa \to 0$ and $\Sigma_c/\Sigma_\kappa \to \infty$, where κ represents h,a,b but not c, cause the COP at the maximum χ merit of figure to approach its upper and lower bounds, $\varepsilon_\chi^- = 0$ and $\varepsilon_\chi^+ = (\sqrt{9+8\varepsilon_C} - 3)/2$, respectively.

When the dissipations in the two adiabatic and two isothermal processes are respectively symmetric, we have $\Sigma_a = \Sigma_b = r\Sigma_h = r\Sigma_c$, with r being the ratio. In such a case we consider three special situations: (i) $r \to 0$. The nonadiabatic dissipations for the two adiabatic processes vanish, while the dissipations during the two isothermal processes are symmetric. Making use of Eq. (24), the Curzon-Ahlborn (CA) COP is recovered, $\varepsilon_\chi^{\Sigma_h = \Sigma_c} = \varepsilon_{CA} = \sqrt{1+\varepsilon_C} - 1$, which is also the upper bound of the COP in this case. (ii) $r \to \infty$. The lower bound of the COP is achieved, $\varepsilon_\chi^- = 0$. (iii)$r = 1$. The dissipations in the four thermodynamic processes are symmetric. Here $\varepsilon_\chi^S \equiv \varepsilon_\chi^*(r=1)$ is defined for convenience, and its value can be obtained numerically based on Eqs. (24) and (27) for any given value of T_h/T_c (i.e., the value of ε_C). At this supersymmetric limit we obtain readily from Eqs. (19) and (20) that the time ratios of τ_κ/τ_c ($\kappa = a,b,h$) are $\tau_\kappa/\tau_c = \sqrt{T_h/(mT_c)}$ with $m = (\varepsilon_\chi^* + 2)/\varepsilon_\chi^*$, and that the time allocations for the remaining three processes are equal ($\tau_h = \tau_a = \tau_b$). In Fig. 2(a) we plot the COP ε_χ^S as a function of ε_C, comparing ε_{CA} with the upper bound ε_χ^+ of the Carnot-like refrigeration cycle.

For an adiabatic process of some heat devices [12], in the sudden limit there is no time for relaxation and heat

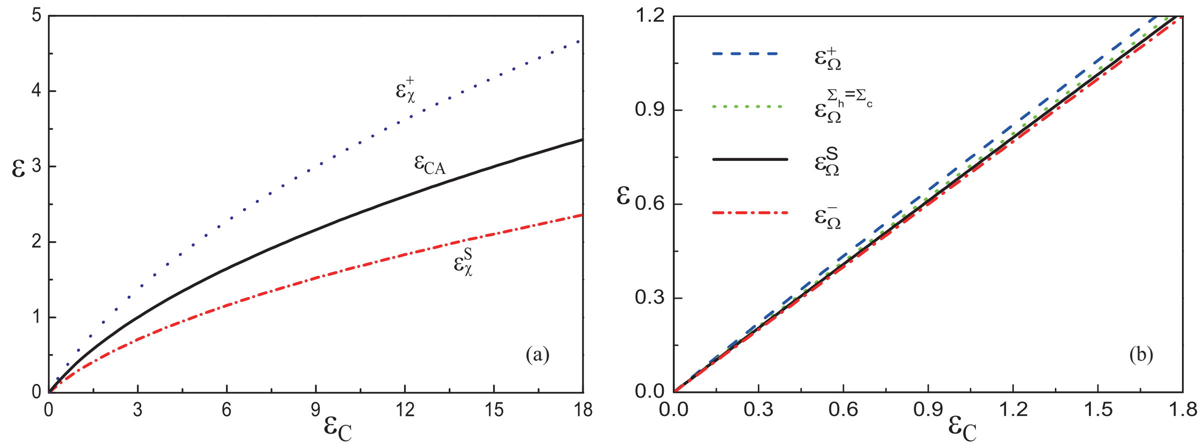

FIG. 2. (Color online) (a) The values of ε_χ in the three limits ε_χ^+ (dotted blue line), ε_{CA} $(=\varepsilon_\chi^{\Sigma_h=\Sigma_c})$ (black solid line), and ε_χ^S (red dot-dashed line) versus the Carnot COP ε_C. (b) The values of ε_Ω in the four cases ε_Ω^+ (blue dashed line), $\varepsilon_\Omega^{\Sigma_h=\Sigma_c}$ (green dotted line), ε_Ω^S (black solid line), and ε_Ω^- (red dot-dashed line) versus the Carnot COP ε_C.

losses to the environment can be minimized. To describe such an adiabatic process for these special systems, we assume that the entropy production in an adiabatic process with vanishing time ($\tau_a = 1/x_a \to 0$ and $\tau_b = 1/x_b \to 0$) is constant and independent of time, while the low-dissipation assumption holds well for an isothermal process. That is, we set $L_a = \Sigma_a$ and $L_b = \Sigma_b$ with Σ_a and Σ_b being constants independent of time, and we have $L'_a = L'_b = 0$. Then the expressions for α_1 and α_2 below Eq. (23) become $\alpha_1 = \frac{\Sigma_c x_c + \Sigma_h x_h}{\Sigma_a+\Sigma_b+\Sigma_c x_c+\Sigma_h x_h}$ and $\alpha_2 = \frac{\Sigma_c x_c}{\Sigma_a+\Sigma_b+\Sigma_c x_c+\Sigma_h x_h}$. For the strong-nonadiabatic-dissipation limit, $\Sigma_a \to \infty$ or $\Sigma_b \to \infty$, we find from Eq. (22) that the COP at the maximum χ figure of merit, ε_χ^*, becomes vanishing, since α_1 as well as α_2 tends to zero. On the contrary, when $\Sigma_a = 0$ and $\Sigma_b = 0$, our result is reduced to the one derived in the idealized nonadiabatic dissipation model, as discussed above. Therefore, if the entropy production in the adiabatic process is constant and independent of time, the lower and upper bounds of ε_χ^* are also given by Eq. (26) which was derived under the assumption that $L_\kappa = \Sigma_\kappa x_\kappa$ with $\kappa = a,b,c,h$.

B. COP at maximum $\dot{\Omega}$ figure of merit

The Ω criterion, a trade-off between maximum cooling and lost cooling loads, is defined as $\Omega = (2\varepsilon - \varepsilon_{\max})W$ [23]. The target function $\dot{\Omega} = (2\varepsilon - \varepsilon_{\max})\frac{W}{\tau_{\text{cycle}}}$ can be expressed as

$$\dot{\Omega} = [2Q_c - \varepsilon_C(Q_h - Q_c)]x_a x_b x_c x_h / (x_a x_b x_c + x_a x_b x_h + x_b x_c x_h + x_a x_c x_h), \tag{28}$$

where we have made the variable transformation $x_\kappa = 1/\tau_\kappa$ $(\kappa = h,c,a,b)$. Setting the derivatives of $\dot{\Omega}$ with respect to x_κ $(\kappa = h,c,a,b)$ equal to zero, we derive the optimal equations

$$[2Q_c - \varepsilon_C(Q_h - Q_c)]\frac{x_b x_c x_h}{x_a} = T_h L'_a \varepsilon_C (x_a x_b x_c + x_a x_b x_h + x_b x_c x_h + x_a x_c x_h), \tag{29}$$

$$[2Q_c - \varepsilon_C(Q_h - Q_c)]\frac{x_a x_c x_h}{x_b} = T_h L'_b \varepsilon_C (x_a x_b x_c + x_a x_b x_h + x_b x_c x_h + x_a x_c x_h), \tag{30}$$

$$[2Q_c - \varepsilon_C(Q_h - Q_c)]\frac{x_a x_b x_c}{x_h} = T_h L'_h \varepsilon_C (x_a x_b x_c + x_a x_b x_h + x_b x_c x_h + x_a x_c x_h), \tag{31}$$

$$[2Q_c - \varepsilon_C(Q_h - Q_c)]\frac{x_a x_b x_h}{x_c} = T_c L'_c (2+\varepsilon_C)(x_a x_b x_c + x_a x_b x_h + x_b x_c x_h + x_a x_c x_h). \tag{32}$$

Dividing Eq. (32) by Eqs. (29), (30), and (31), respectively, we have

$$\frac{x_c}{x_a} = \sqrt{\frac{L'_a(1+\varepsilon_C)}{L'_c(2+\varepsilon_C)}}, \tag{33}$$

$$\frac{x_c}{x_b} = \sqrt{\frac{L'_b(1+\varepsilon_C)}{L'_c(2+\varepsilon_C)}}, \tag{34}$$

$$\frac{x_c}{x_h} = \sqrt{\frac{L'_h(1+\varepsilon_C)}{L'_c(2+\varepsilon_C)}}. \tag{35}$$

It follows, by substitution of $\tau_\kappa = 1/x_\kappa (\kappa = h,c,a,b)$ into Eqs. (33), (34), and (35), that the optimal ratios of the time $\tau_\kappa/\tau_h (\kappa = a,b)$ as well as τ_b/τ_a are still given by Eq. (19), but that under the Ω criterion the time ratio $\tau_\kappa/\tau_c (\kappa = h,a,b)$ becomes

$$\frac{\tau_\kappa}{\tau_c} = \sqrt{\frac{L'_\kappa(1+\varepsilon_C)}{L'_c(2+\varepsilon_C)}} \quad (\kappa = a,b,h). \tag{36}$$

Directly adding both sides of Eqs. (29), (30), (31), and (32), we obtain

$$2Q_c - \varepsilon_C(Q_h - Q_c) = T_h(L'_a x_a + L'_b x_b + L'_h x_h)\varepsilon_C + T_c L'_c x_c (2+\varepsilon_C). \tag{37}$$

Substitution of Eqs. (6) and (7) into Eq. (37) leads to

$$\Delta S = (2+\varepsilon_C)(L_c + L'_c x_c) + (1+\varepsilon_C)(L_a + L_b + L_h + L'_a x_a + L'_b x_b + L'_h x_h). \tag{38}$$

It follows, after substituting Eq. (38) into Eq. (10), that the COP for the maximum $\dot{\Omega}$ condition is

$$\varepsilon_{\Omega}^{*}=\frac{(2+\varepsilon_C)L_c'x_c+(1+\varepsilon_C)(L_a+L_b+L_h+L_a'x_a+L_b'x_b+L_h'x_h+L_c)}{(2+\varepsilon_C)L_c'x_c+2(1+\varepsilon_C)(L_a+L_b+L_h+L_c)+(1+\varepsilon_C)(L_a'x_a+L_b'x_b+L_h'x_h)}\varepsilon_C. \tag{39}$$

Assuming that the irreversible entropy production for an isothermal or an adiabatic step is inversely proportional to the time for completing that process, i.e., $L_\kappa'=\Sigma_\kappa$ $(\kappa=a,b,c,h)$, Eq. (39) becomes

$$\varepsilon_{\Omega}^{*}=\frac{3+2\varepsilon_C+2\gamma}{4+3\varepsilon_C+3\gamma}\varepsilon_C, \tag{40}$$

where $\gamma=\sqrt{(1+\varepsilon_C)(2+\varepsilon_C)\Sigma_a/\Sigma_c}+\sqrt{(1+\varepsilon_C)(2+\varepsilon_C)\Sigma_b/\Sigma_c}+\sqrt{(1+\varepsilon_C)(2+\varepsilon_C)\Sigma_h/\Sigma_c}$, which simplifies to $\gamma=\sqrt{(1+\varepsilon_C)(2+\varepsilon_C)\Sigma_h/\Sigma_c}$ in the ideal adiabatic refrigeration cycle, with use of Eqs. (33), (34), and (35). The value of γ is a non-negative number, varying from 0 to ∞. Hence, the COP at the maximum $\dot{\Omega}$ figure of merit, ε_{Ω}^{*}, must be situated in the range

$$\varepsilon_{\Omega}^{-}\equiv\frac{2}{3}\varepsilon_C\leqslant\varepsilon_{\Omega}^{*}\leqslant\frac{3+2\varepsilon_C}{4+3\varepsilon_C}\varepsilon_C\equiv\varepsilon_{\Omega}^{+}. \tag{41}$$

The upper and lower bounds for the optimized COP at the maximum $\dot{\Omega}$ figure of merit, ε_{Ω}^{-} and ε_{Ω}^{+}, versus the Carnot COP ε_C, are plotted in Fig. 2(b).

As in the case of the χ figure of merit, the expression for the COP at the maximum $\dot{\Omega}$ condition is similar to the corresponding one obtained in the model [19] with idealized adiabatic processes, and the internally nonadiabatic dissipation has no influence on the bounds of the COP. Here the optimal value of the COP, however, represents a broader context by including the nonadiabatic dissipation and the time required for completing any adiabat.

If the dissipations of the two adiabatic and two isothermal processes are respectively symmetric, i.e., $\Sigma_a=\Sigma_b=r\Sigma_c=r\Sigma_h$, then $\gamma=(2\sqrt{r}+1)\sqrt{(1+\varepsilon_C)(2+\varepsilon_C)}$, and Eq. (39) becomes

$$\varepsilon_{\Omega}^{*}(r)=\frac{3+2\varepsilon_C+(4\sqrt{r}+2)\sqrt{(1+\varepsilon_C)(2+\varepsilon_C)}}{4+3\varepsilon_C+(6\sqrt{r}+3)\sqrt{(1+\varepsilon_C)(2+\varepsilon_C)}}\varepsilon_C. \tag{42}$$

From Eq. (42), we find in such a case that the bounds of the COP for the maximum $\dot{\Omega}$ figure of merit are achieved, $\frac{2}{3}\varepsilon_C\leqslant\varepsilon_{\Omega}^{*}(r)\leqslant\frac{3+2\varepsilon_C+2\sqrt{(1+\varepsilon_C)(2+\varepsilon_C)}}{4+3\varepsilon_C+3\sqrt{(1+\varepsilon_C)(2+\varepsilon_C)}}\varepsilon_C$, when $r\to\infty$ and $r\to 0$, respectively. In the particular case when the dissipations of the four thermodynamic processes are symmetric, the COP can be obtained by the use of $r=1$,

$$\varepsilon_{\Omega}^{S}\equiv\varepsilon_{\Omega}^{*}(r=1)=\frac{3+2\varepsilon_C+6\sqrt{(1+\varepsilon_C)(2+\varepsilon_C)}}{4+3\varepsilon_C+9\sqrt{(1+\varepsilon_C)(2+\varepsilon_C)}}\varepsilon_C. \tag{43}$$

Then the optimal time ratio of $\tau_\kappa/\tau_c(\kappa=h,a,b)$ in Eq. (36) simplifies to $\tau_\kappa/\tau_c=\sqrt{T_c/(2T_h-T_c)}(\kappa=h,a,b)$ in this superasymmetric case, while the optimized times spent on the other three processes are equal ($\tau_a=\tau_b=\tau_h$). At the supersymmetric limit, the time ratios of τ_κ/τ_ν with $\kappa=h,a,b$ and $\nu=h,c$ as functions of the Carnot COP ε_C, under the Ω and χ criteria, are plotted in Fig. 3 by using Eqs. (19), (20), and (36). Figure 3 shows that, whether under the χ or Ω criterion, the time taken for the cold isothermal process is larger than the ones for the other three processes, on which the times spent are equal to each other. This result is in contrast to the fact that, for an irreversible heat engine [29], the hot isothermal process proceeds most slowly during a cycle, with equal times required for completing the cold isothermal and two adiabatic processes. This is not surprising, since the heat is transported into the system during the cold (hot) isothermal process for the refrigerator (heat engine), and the additional heat developed by the nonadiabatic dissipation is related to the high temperature T_h (low temperature T_c) for the refrigerator (heat engine).

If an adiabatic process is ideal and thus isentropic ($L_a=L_b=0$), for low-dissipation refrigerators with $L_c'=\Sigma_c$ and $L_h'=\Sigma_h$, the symmetric limit ($\Sigma_c=\Sigma_h$) gives rise to the following form of Eq. (39):

$$\varepsilon_{\Omega}^{\Sigma_c=\Sigma_h}\equiv\varepsilon_{\Omega}^{*}(\Sigma_a=\Sigma_b=0,\Sigma_h=\Sigma_c)=\frac{3+2\varepsilon_C+2\sqrt{(1+\varepsilon_C)(2+\varepsilon_C)}}{4+3\varepsilon_C+3\sqrt{(1+\varepsilon_C)(2+\varepsilon_C)}}\varepsilon_C, \tag{44}$$

which can be simplified as

$$\varepsilon_{\Omega}^{\Sigma_c=\Sigma_h}=\frac{\varepsilon_C}{\sqrt{(1+\varepsilon_C)(2+\varepsilon_C)}-\varepsilon_C}. \tag{45}$$

In deriving Eq. (44), we have used Eq. (35). When the nonadiabatic dissipation is negligible and the low dissipations for the two isothermal processes are symmetric, the COP at maximum $\dot{\Omega}$ as given in Eq. (45) is identical to the COP for

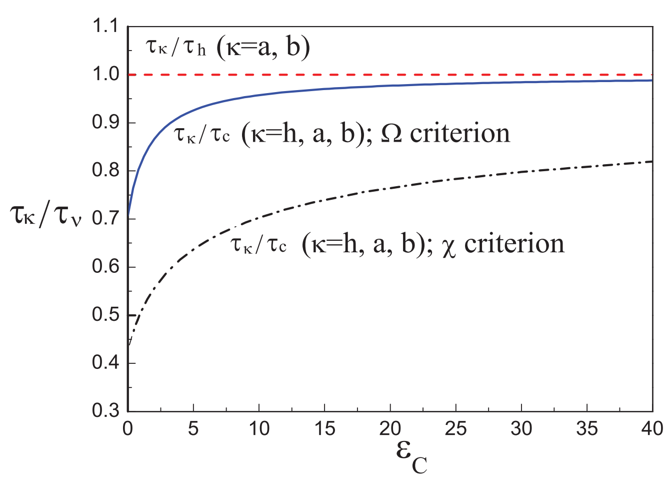

FIG. 3. (Color online) The ratios of τ_κ/τ_ν ($\kappa=h,a,b$, and $\nu=h,c$) within the maximum χ as well as the $\dot{\Omega}$ figure of merit versus the Carnot COP ε_C at the supersymmetric limit. Here the optimal values of τ_κ/τ_c ($\kappa=h,a,b$) under the χ and Ω criteria, are indicated by a black dash-dotted line and a blue solid line, respectively. The values of τ_κ/τ_h ($\kappa=a,b$) are equal to 1 under both χ and Ω criteria, and are represented by a red dashed line.

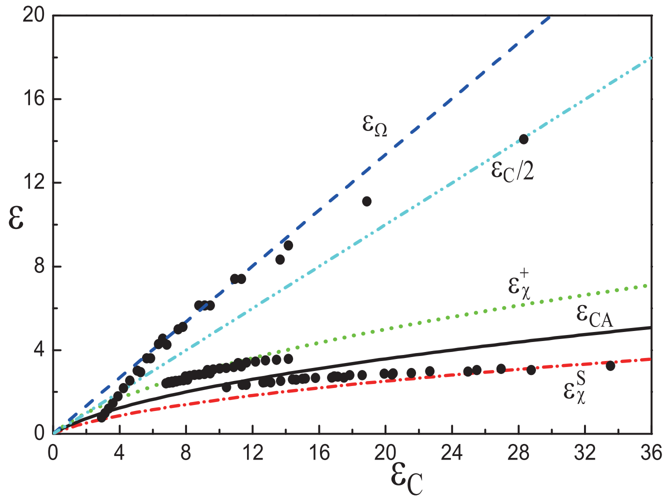

FIG. 4. (Color online) Comparison between theoretical results (lines) and three sets of experimental results (points). Here ε_Ω, $\varepsilon_C/2$, ε_χ^+, ε_{CA}, and ε_χ^S are indicated by a blue dashed line, a cyan dot-dot-dashed line, a green dotted line, a black solid line, and a red dot-dashed line, respectively. The values in the four cases of ε_Ω^*, ε_Ω^+, $\varepsilon_\Omega^{\Sigma_h=\Sigma_c}$, ε_Ω^S, and ε_Ω^-, are indistinguishable and collapse into a single curve on this plotted scale and thus all of these values are denoted by ε_Ω.

Carnot-like refrigerators with the same optimization criterion but within the finite-time thermodynamics context and under the endoreversible assumption [23]. At the symmetric limits (either with or without nonadiabatic dissipation) for the optimal COPs, ε_Ω^S determined according to Eq. (43) and $\varepsilon_\Omega^{\Sigma_c=\Sigma_h}$ given by Eq. (45), are also shown in Fig. 2(b). It is clear from Fig. 2(b) that the nonadiabatic dissipation leads to a very slight decrease in the COP.

The irreversible entropy production for the instantaneous adiabatic process in which $\tau_a = 1/x_a \to 0$ and $\tau_b = 1/x_b \to 0$ may be assumed to be constant and independent of time, while low dissipation is adopted to describe the irreversible isothermal process, because of the fact pointed out in Sec. III A. That is, $L_a = \Sigma_a$ and $L_b = \Sigma_b$, while $L_c = \Sigma_c x_c$ and $L_h = \Sigma_h x_h$, with $\Sigma_\kappa(\kappa = a,b,c,h)$ being constants independent of time. Substituting these relations into Eq. (39), we find that the value of $\varepsilon_C/2$ as the lower bound of ε_Ω^* is achieved when $\Sigma_a \to \infty$ or $\Sigma_b \to \infty$. Thus the values of ε_Ω^* are situated in the range $\varepsilon_C/2 \leqslant \varepsilon_\Omega^* \leqslant \varepsilon_\Omega^+$, where the upper bound Ω^+, defined by Eq. (41), can be achieved for the two isentropic adiabatic processes ($\Sigma_a = \Sigma_b = 0$).

Note that even the lower bound of COP under the Ω criterion is finite and considerably larger than the upper bound obtained under the χ optimization criterion, as shown in Fig. 4. It is therefore indicated that, in comparison with the χ criterion, the objective function $\dot{\Omega}$ can be adopted as one guide to design more efficient refrigerators.

C. Comparison between our prediction and experimental data

It would be instructive to compare our theoretical predictions with the observed COPs of some real refrigerators. Our theoretical prediction versus the data for real refrigerators [37] at different values of temperature are plotted in Fig. 4, which shows that the theoretical results agree well with the experimental refrigerator data, whether at maximum χ or maximum $\dot{\Omega}$ figure of merit. In the case when the low-dissipation assumption is valid for the isothermal as well as the adiabatic processes, applying the Ω criterion to optimization of the refrigerator cycle, we find that there are relatively small differences even between the lower and upper bounds (ε_Ω^+ and ε_Ω^-) of the COP for the refrigerator cycle. If the entropy production in the adiabatic process is assumed to be constant and independent of time, while the low-dissipation assumption is valid for the two isothermal processes, the bounds of ε_Ω^* are given by $\varepsilon_C/2 \leqslant \varepsilon_\Omega^* \leqslant \varepsilon_\Omega^+$. The values of ε_Ω^*, ε_Ω^+,$\varepsilon_\Omega^{\Sigma_h=\Sigma_c}$, ε_Ω^S, and ε_Ω^- are indistinguishable in the plotted scale of Fig. 4, where we use ε_Ω to represent these values. These values, together with the value of $\varepsilon_C/2$, are in good agreement with experimental data, particularly for some values of ε_C. Under the maximum χ condition, our calculation of the COP in the symmetric limit, ε_χ^S, matches more closely with the experimental data than the corresponding ones obtained in the previous model with idealized adiabatic processes, $\varepsilon_{CA} = \varepsilon_\chi^{\Sigma_h=\Sigma_c}$, as expected. Hence, our result suggests that internally nonadiabatic dissipation indeed induces effects on the performance of heat devices and thus cannot be considered negligible in comparisons with the experimental data.

IV. CONCLUSION

In conclusion, we have analyzed the COP with the χ and $\dot{\Omega}$ figures of merit for an irreversible Carnot-like refrigerator with nonadiabatic dissipation, assuming that the irreversible entropy production during the isothermal process is inversely proportional to the time required for completing this process. In the case when the low-dissipation condition holds well not only in the isothermal but also in the adiabatic processes, we have found the following. (i) The limits of extremely asymmetric dissipation lead to the fact that the COP, either under the χ or under the Ω criterion, converges to the same bounds as the corresponding ones obtained from previous models with idealized adiabatic processes. (ii) When the dissipations in the two isothermal and two adiabatic processes are respectively symmetric, comparison between our theoretical predictions of the COP with the maximum χ figure of merit and the observed COPs of real refrigerators shows that our values match the experimental results more closely than the ones derived in previous models with no inclusion of nonadiabatic dissipation. If the nonadiabatic dissipation for the instantaneous adiabatic process is describe as a constant independent of time, we have shownthe following. (1) The bounds of the COP under the χ criterion are the same as those obtained in the case when the nonadiabatic dissipation is depicted using the low-dissipation assumption. (2) In the strong limit of nonadiabatic dissipation, the lower bound of the COP with the maximum $\dot{\Omega}$ figure of merit becomes equal to $\varepsilon_C/2$, while its upper bound is reduced to that for the previous model without nonadiabatic dissipation.

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APPENDIX: DERIVATION OF EQUATION (22)

Using Eq. (8), we can rewrite Eq. (21) as

$$\frac{(Q_h - Q_c)(L_a + L_b + L_h + L_c)}{Q_h/T_h - Q_c/T_c} = \frac{2Q_h T_c L_c' x_c - T_c L_c' x_c Q_c + Q_c T_h(L_a' x_a + L_b' x_b + L_h' x_h)}{Q_c} \tag{A1}$$

or

$$\frac{T_h T_c (Q_h - Q_c)(L_a + L_b + L_h + L_c)}{Q_h T_c - Q_c T_h} = \frac{2Q_h T_c L_c' x_c - T_c L_c' x_c Q_c + Q_c T_h(L_a' x_a + L_b' x_b + L_h' x_h)}{Q_c}. \tag{A2}$$

Equation (A2), together with $\varepsilon_\chi^* = \frac{Q_c}{Q_h - Q_c}$ and $\varepsilon_C = \frac{T_c}{T_h - T_c}$, can be used to derive Eq. (A3):

$$\begin{aligned} \frac{1}{\varepsilon_\chi^*} - \frac{1}{\varepsilon_C} &= \frac{T_h(L_a + L_b + L_c + L_h)(Q_h - Q_c)}{2Q_h T_c L_c' x_c - T_c L_c' x_c Q_c + Q_c T_h(L_a' x_a + L_b' x_b + L_h' x_h)} \\ &= \frac{1}{\frac{2T_c L_c' x_c (Q_h - Q_C) + T_h L_c' x_c Q_c + Q_c T_h (L_a' x_a + L_b' x_b + L_h' x_h) - L_c' x_c Q_c (T_h - T_c)}{T_h (L_a + L_b + L_c + L_h)(Q_h - Q_c)}} \\ &= \frac{1}{\frac{2T_c(T_h - T_c)}{T_h(T_h - T_c)} \frac{L_c' x_c}{(L_a + L_b + L_h + L_c)} + \frac{T_h Q_c (L_a' x_a + L_b' x_b + L_h' x_h + L_c' x_c) - Q_c L_c' x_c (T_h - T_c)}{T_h (L_a + L_b + L_h + L_c)(Q_h - Q_c)}}. \end{aligned} \tag{A3}$$

After some simple reshuffling, Eq. (A3) can be expressed in the form

$$\begin{aligned} \frac{1}{\varepsilon_\chi^*} - \frac{1}{\varepsilon_C} &= \frac{1}{\left[\frac{2T_c(T_h - T_c)}{T_h(T_h - T_c)} - \frac{Q_c(T_h - T_c)}{T_h(Q_h - Q_c)}\right] \frac{L_c' x_c}{(L_a + L_b + L_h + L_c)} + \varepsilon_\chi^* \frac{L_a' x_a + L_b x_b' + L_c' x_c + L_h' x_h}{(L_a + L_b + L_h + L_c)}} \\ &= \frac{1}{\varepsilon_\chi^* \frac{L_a' x_a + L_b x_b' + L_c' x_c + L_h' x_h}{(L_a + L_b + L_h + L_c)} + \frac{2\varepsilon_C - \varepsilon_\chi^*}{1 + \varepsilon_C} \frac{L_c' x_c}{(L_a + L_b + L_h + L_c)}}. \end{aligned} \tag{A4}$$

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