Uncertainties in Uranium and Uranium-235 determination in fuel rods with multizones

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Introduction

The fuel assemblies for Light Water Reactor are made up with fuel rods which can contain several zones. Each zone is characterized by the use of the same type of pellets. For example: Some kinds of fuel rods for assemblies GE12 or GE14 have 5 zones: two of natural uranium at the bottom and the top, two intermediate ones exclusively with enriched UO_2 and a central zone of UO_2 and gadolinium. For accountancy purposes (EURATOM and IAEA inventory, uranium offset with customers, etc.) it is necessary to know the contents of U and U-235 in each zone and their corresponding uncertainties.

The U and U-235 can be calculated by weighing the column in pellets of each zone before loading it in the fuel rod. It is a very slow process and its automation is quite complicated. At *Fabrica de Juzbado* we apply a method that involves weighing the loaded fuel rod and measuring the length of each zone with a scanner. Once this is estimated, the assignment of the weight of Uranium and Uranium-235 should be simple. However, many variables contribute to the determination of the U and U-235. It is very important to evaluate their influence in the measurements (uncertainties). For instance, in a reload of 30 tU a difference of 0.1% in the determination of U is equivalent to 30 kg U whose economic value could as much as $30000 or let us take a Nuclear Fuel Fabrication Plant that produces 300 tU a year, a systematic error of 0.1% in U would be equivalent to 300 kg of U in the difference of the annual inventory. We will describe the procedure applied to evaluate the uncertainties and their values identifying what error sources can be relevant.

The evaluation of Uncertainties

The weight of Uranium, WU_j, in the j-zone of fuel rods formed by m zones, is evaluated by applying the eq(1) and the weight of uranium 235, We_j, by using the eq(2).

$$WU_j = \frac{N\,U_j\,l_j\,\rho_j}{\sum_j^m l_j\,\rho_j} \qquad (1)$$

$$We_j = \frac{N e_j U_j l_j \rho_j}{\sum_j^m l_j \rho_j} \qquad (2)$$

where:

- N Net weight of the fuel rods. It is obtained by the difference between the gross weight of the fuel rod and the weight of the tube (with neither end plug nor spring).
- U_j Concentration of uranium of the pellets of the zone j. To measure it, we use gravimetric analysis for UO_2 pellets and X-Ray fluorescence for gadolinium pellets.
- e_j Concentration (enrichment) of U-235 in the uranium of the zone j. It is measured by means of mass spectrometry.
- ρ_j Density in the j zone measured by a geometric method.
- l_j Length of the j zone measured with a scanner. In the case of fuel rods without gadolinium, the enrichment is measured with INaTl detectors once the fuel rods have been activated neutronically with Cf-252. Fuel rods with gadolinium pellets are

measured by using BGO detectors. The scanner measures the length L_j between the top of j zone and the bottom of zone 1. The true length of each zone j, denoted by l_j, is given by $l_1 = L_1$, $l_2 = L_2 - L_1$,..., $l_j = L_j - L_{j-1}$.

The measurement of each one of previous variables has its uncertainties. In these cases, a measured y is determined from other quantities $x_1, x_2, \ldots, x_K$ through a functional relationship $y = f(x_1, x_2, \ldots, x_K)$. The combined standard uncertainty of the estimated y, denoted by $u_c(y)$, is the positive root of the combined variance which is given by:

$$u_c^2(y) = \sum_{i=1}^{K} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) \qquad (3)$$

This expression is obtained from the first order terms of a Taylor series expansion. It is an approximation when function is linear and all the quantities are independent (For a more detailed discussion see /1/).

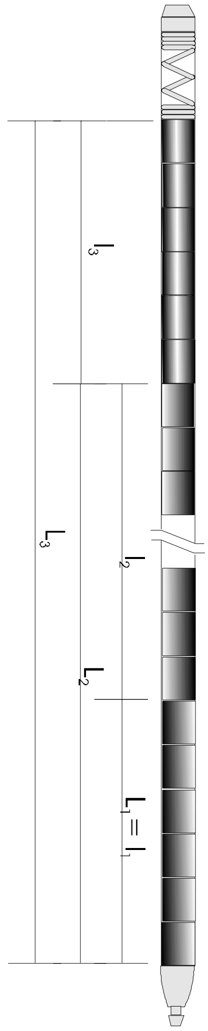

Fig. 1.- Fuel rod with three zones

We are going to assess the uncertainties for the case of fuel rods with 3 zones (*Fig. 1*), which is the most frequent in the multizones ones. The describe procedure can be used for rods with any number of zones. We will start by evaluating the uncertainty for the second zone (j = 2). The procedure is the same for zones 1 and 3.

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According with eq(1) and bearing in mind that the length of a zone is $l_j = L_j - L_{j-1}$, the net weight, W_j, in zone 2 for a 3-zone fuel rod is given by

$$W_2 = \frac{N(L_2 - L_1)\rho_2}{L_1\rho_1 + (L_2 - L_1)\rho_2 + (L_3 - L_2)\rho_3} \qquad (4)$$

From ec(4) we deduce that the uncertainty, u_{Wj}, in the weight of j zone is a function of 5 variables: N, L_1, L_2, L_3, ρ_1, ρ_2, ρ_3. The uncertainty in the weight of zone 2 and it is given by eq(5). We have denoted the uncertainty in the measure of each variable by u_N, u_{L1}, u_{L2}, u_{L3}, $u_{\rho 1}$, $u_{\rho 1}$, $u_{\rho 2}$, $u_{\rho 3}$ and also σ_N, σ_{Lj}, $\sigma_{\rho j}$ (j = 1, 2, 3), which are the standard deviations in the determination of net weight, length and density.

$$u_{W2}^2 = u_N^2 + u_{L1}^2 + u_{L2}^2 + u_{L3}^2 + u_{\rho 1}^2 + u_{\rho 2}^2 + u_{\rho 3}^2 \quad (5)$$

where:

$$u_N^2 = \sigma_N^2 \frac{(L_1 - L_2)^2 \rho_2^2}{\left(L_1(\rho_1 - \rho_2) + L_2(\rho_2 - \rho_3) + L_3\rho_3\right)^2}$$

$$u_{\rho_1}^2 = \sigma_{\rho_1}^2 \frac{N^2 L_1^2 (L_2 - L_1)^2 \rho_2^2}{\left(L_1(\rho_1 - \rho_2) + L_2(\rho_2 - \rho_3) + L_3\rho_3\right)^4}$$

$$u_{\rho_2}^2 = \sigma_{\rho_2}^2 \frac{N^2 (L_2 - L_1)^2 \left(L_1\rho_1 + (L_3 - L_2)\rho_3\right)^2}{\left(L_1(\rho_1 - \rho_2) + L_2(\rho_2 - \rho_3) + L_3\rho_3\right)^4}$$

$$u_{\rho_3}^2 = \sigma_{\rho_3}^2 \frac{N^2 (L_2 - L_1)^2 (L_3 - L_2)^2 \rho_2^2}{\left(L_1(\rho_1 - \rho_2) + L_2(\rho_2 - \rho_3) + L_3\rho_3\right)^4}$$

$$u_{L_1}^2 = \sigma_{L_1}^2 \frac{N^2 \rho_2^2 \left(L_2(\rho_1 - \rho_3) + L_3\rho_3\right)^2}{\left(L_1(\rho_1 - \rho_2) + L_2(\rho_2 - \rho_3) + L_3\rho_3\right)^4}$$

$$u_{L_2}^2 = \sigma_{L_2}^2 \frac{N^2 \rho_2^2 \left(L_1(\rho_1 - \rho_3) + L_3\rho_3\right)^2}{\left(L_1(\rho_1 - \rho_2) + L_2(\rho_2 - \rho_3) + L_3\rho_3\right)^4}$$

$$u_{L_3}^2 = \sigma_{\rho_3}^2 \frac{N^2 (L_2 - L_1)^2 \rho_2^2 \rho_3^2}{\left(L_1(\rho_1 - \rho_2) + L_2(\rho_2 - \rho_3) + L_3\rho_3\right)^4}$$

In the practice we can employ the following approximations: $\sigma_{L1} \approx \sigma_{L2} \approx \sigma_{L3} \approx \sigma_L$, $\rho_1 \approx \rho_2 \approx \rho_3$, $\sigma_{\rho 1} \approx \sigma_{\rho 2} \approx .\sigma_{\rho} . \approx .\sigma_{\rho 3}$, then we obtain (6)

$$u_{W2}^2 = K_{N2}^2 \sigma_N^2 + K_{L2}^2 \sigma_L^2 + K_{\rho 2}^2 \sigma_\rho^2 \quad (6)$$

where:

$$K_{N2}^2 = \frac{(L_2 - L_1)^2}{L_3^2}$$

$$K_{L2}^2 = 2N^2 \left(\frac{1}{L_3^2} + \frac{(L_2 - L_1)^2}{L_3^4} \right)$$

$$K_{\rho 2}^2 = \frac{2N^2 (L_2 - L_1)^2}{L_3^4 \rho^2} [(L_1^2 + (L_3 - L_2)^2 + L_1(L_3 - L_2)]$$

Substituting $W_j = (N\, l_j\, \rho_j) / \sum_j^m l_j\, \rho_j$ in eq(1) and eq(2) we obtain the Uranium and Uranium-235 net weight in zone j, denoted by WU_j and We_j, respectively

$$WU_j = U_j\, W_j \quad (7)$$

$$We_j = e_j\, U_j\, W_j \quad (8)$$

The uncertainties for this kind of equations are given by

$$u_c^2(y) = \hat{y}^2 \sum_1^n \left(\frac{u_i}{\hat{x}_i} \right)^2 \quad (9)$$

where $\hat{y}$ and $\hat{x}_i$ are the measurement average of y and x_i values, respectively.

Then, the uncertainty in the estimation of Uranium net weight u_{WUj}, in zone j is evaluated by

$$u^2_{WU_j} = \langle WU \rangle^2 \left[\left(\frac{u_{Wj}}{W_j} \right)^2 + \left(\frac{u_{Uj}}{U_j} \right)^2 \right] \quad (10)$$

where u_{Uj} is the uncertainty in the determination of Uranium concentration of zone j and u_{Wj} is the uncertainty in the weight of zone j. In the case j=2, u_{W2} is obtained from eq(6).

In the same way, uncertainty in the estimation of Uranium-235 in zone j is given by eq(11), where u_{ej} is the uncertainty in the enrichment measurement of zone j.

$$u^2_{We_j} = \langle We \rangle^2 \left[\left(\frac{u_{UWj}}{UW_j} \right)^2 + \left(\frac{u_{ej}}{e_j} \right)^2 \right] \quad (11)$$

Example

Typical GE12 fuel rod values are:
N = 2400 g, L_3 = 380 cm, L_2 = 300, L_1=50 cm, ρ= 10.5 g cm^{-3} , σ_N = 0.5 g, σ_L= 0.25 cm, σ_ρ = 0.02 g cm^{-3}, U(concentration) = 0.880, σ_U= 0.001, e(enrichment) = 0.0400, σ_e =0.00005, then

$$u^2_{W2} = 97.4\,\sigma^2_L + 0.43\,\sigma^2_N + 4057.1\,\sigma^2_\rho$$

where the main contribution is the one of first term, σ_L (78%) and the next σ_ρ. (21%). The other term σ_N is insignificant and it could be eliminated. For the data mentioned before the relative uncertainties are:

$$\frac{u_{W2}}{W2} \approx 0.18\% \quad \text{(Net weight)}$$

$$\frac{u_{WU2}}{WU_2} \approx 0.21\% \quad \text{(Uranium weight)}$$

$$\frac{u_{We2}}{We_2} \approx 0.24\% \quad \text{(Uranium 235 weight)}$$

These are the uncertainties in a fuel rod. In a fuel assembly, with n rods, the uncertainties will be smaller because we will have to divide the random error by $\sqrt{n}$.

Conclusion

To estimate the U and U-235 in multizones fuel rods at *Fabrica de Juzbado* we apply a method by weighing the loaded fuel rod and measuring the length of each zone with a scanner. The uncertainties of the measurement are small, and in the fuel assemblies these are negligible.

Reference

/1/.ISO. Guide to the expression of uncertainly in measurement. Expression of Uncertainty: 1993 (E).)