Male Investment in Schooling with Frictional Labour and Marriage Markets

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Abstract

We present an equilibrium model with inter-linked frictional labour and marriage markets. Women's flow value of being single is treated as given, and it captures returns from employment (possibly augmented by investment in education). Single unemployed men conduct a so-called constrained sequential job search, and can choose to improve their labour market returns *as well as* their marriage prospects by undertaking a costly ex-ante investment in schooling that leads to higher wage offers and hence potential marriage. We establish the existence of a market equilibrium where only a fraction of men choose to get educated, and show that this proportion decreases if there is an increase in womens' labour market returns (educational attainment). We also address the conditions for the stability of such equilibrium.

Keywords: frictional markets, constrained search, male schooling

1 Introduction

One of the most puzzling recent trends has been the apparent decrease in male educational attainment, despite an increase in labour market returns. In the last few decades, female education enrollment has been increasing, whereas men's college graduation rates have decreased or stagnated. In an important study, Goldin et al. (2006) found that although women represented only

39% of U.S. undergraduates in 1960, within four decades they made up the majority of U.S. college students and of those graduating with a bachelor's degree. The trend is by no means limited to the U.S.. The same study reported that, while school enrollment rates of women in 17 OECD countries were below those of men in the mid-80's, by 2002 women's college enrollment rates exceeded those of men in 15 of these countries.

We provide an equilibrium model of inter-linked frictional labour and marriage markets, with focus on men's choice of schooling investment. Education enhances not only the labour market returns of men (through improved wage offers) but also, indirectly, their marriage prospects - through their earned wages. We investigate the existence and properties (including stability) of a market equilibrium where some men choose to invest in education and women are selective about whom they marry. In particular, we show that an increase in single women's labour market options (viewed as a proxy for their educational attainment) leads to a decrease in the equilibrium fraction of men who invest in schooling.

Our model is underpinned by the well known fact that couples tend to sort according to various traits, such as education.[1]Here, as all men are ex-ante homogeneous, the only relevant differentiating trait they bring to the marriage market is their wage, determined to some extent by the educational choice. Furthermore, since the study of equilibrium class formation is not one of the objectives of the present paper, the sorting aspect in our frictional marriage market essentially boils down to women having an reservation strategy. This leads to two important modelling choices.

First, with earned wages being the only distinguishing male trait in the marriage market, a single woman will only accept a single employed man if his wage is higher than a threshold wage. Consequently, a single unemployed man is involved in a so-called *constrained* sequential job search problem, whereby his marriage market prospects (marriageability) depend on his earnings, and therefore he needs to adjust his labour market strategy accordingly. Hence, before being able to consider the question of schooling investment, one has to characterise the optimal reservation wage policy of an unemployed single man, which is now a function of the female reservation wage. Second, in order to obtain a meaningful female reservation wage in

[1]See Becker (1991).

the frictional marriage market, we include as a *parameter* the flow utility of being single for a woman, which is meant to capture her labour market options and returns - possibly augmented through educational investment. Following Bonilla et. al (2019), we assume that upon marriage women give up this flow value, there is no intra-marital bargaining, and the wage earned by the man becomes a public good for the two of them.

A couple of comments about the latter assumptions. Given our focus on the effect of an increase in female education on the fraction of men who invest in schooling, in this paper we choose to proxy the labour market returns of women with their flow utility outside marriage - just like in Blau et al. (2000). But this is of course a much more general parameter that could also be interpreted as a measure of women's (and, by comparison, men's) attitudes towards marriage, and therefore able to capture possible asymmetries in terms of how they perceive the gains from marriage.[2] The assumption that couples do not negotiate over the surplus from marriage simplifies the analysis: it does not qualitatively affect the nature of men's constrained job search decision, and it allows us to ignore the well-understood question of inefficiency in frictional markets due to the hold-up problem.

Given this framework, our main theoretical result that an increase in female labour market returns (educational attainement) decreases the proportion of men who invest in education has the following intuitive explanation. In essence, we have a two-stage game in which first the single unemployed men choose whether or not to to invest in schooling, this being followed by the interaction in the joint labour and marriage markets. In the latter subgame women and men simultaneously set their reservation wages, women taking as given the fraction of educated men. The partial search equilibrium in the inter-linked frictional markets determines the returns to education for men, both in terms of wages and marriage prospects. For any given cost of schooling, and with a binary education decison problem, a market equilibrium in which a fraction of men choose to undertake the schooling investment requires that the returns to education equal the cost of education. Crucially, it is the proportion of educated men that adjusts so that this market equilibrium condition holds. To see this, consider an increase in female labour market returns. Women become pickier in the marriage market

[2]For example, the empirical results in Gould and Paserman (2003) suggest that men do not seem to care much about their partner's wage.

and increase their reservation wage, but overall only the partial equilibrium of the joint frictional markets is affected directly. Importantly, with a fixed cost of schooling, the equilibrium returns from education for males need to remain the same. Since a change in the fraction of educated men active in the frictional markets affects the female reservation wage in the same direction as a change in female labour market returns, it follows that if the latter increases, the former needs to decrease in order to restore the steady state market equilibrium condition.

Our result is surprisingly robust and, apart from offering an explanation for the puzzling trend documented in Goldin et al. (2006) and others, it also seems to be in line with several other empirical findings. In terms of women's attitudes in the marriage market, Gould and Paserman (2003) conclude that women are pickier if female wages (their proxy for women's value of being a single) are higher. Similarly, Blau et al. (2000) find in their sample that higher labour market returns for females has lead to lower marriage rates for women between ages 16-24 and 25-34. Finally, Oppenheimer (1988) and Oppenheimer and Lew (1995) argue that an improvement in labour market gains for women leads to them delaying the timing of marriage.

This paper is part of a research agenda whose main message is that many observed outcomes in labour markets (including human capital accumulation) can well be the result of individuals' considerations and expectations in the marriage market - and vice-versa. As such, the present work complements Bonilla and Kiraly (2013) and Bonilla et al. (2019), where the the concept of constrained sequential job search was introduced and analysed in detail. For the specific question of the male/female schooling gap, our work complements the important contribution by Chiappori et al. (2009), who offer an explanation that also stresses the link between the marriage market and labour market.[3] However, their setup is completely different from ours, as they consider a frictionless environment, stable marriage assignments and transferable utility within couples, and the focus is essentially on what determines women's educational choice, and when would it be likely to lag behind or overtake that of men's. In terms of models with a frictional labour market, Flinn and Mullins (2015) introduce endogenous productivity-enhancing schooling in a Pissarides (2000) type general equilibrium model augmented with on-the-job search and potential wage renegotiations.

[3] See also Browning et al. (2014).

2 The Model

The economy consists of a continuum of men and women. All agents are risk neutral and have discount rate r. We only consider steady state equilibria.

Men enter the economy unemployed, single and of type L. The distribution of wages faced by them is $F_L(.)$ with support $[\underline{w}_L, \overline{w}_L]$. They can decide to enter the labour market immediately, or choose to undertake an investment in education at a given cost c, same for all men. A man who undertakes the schooling investment becomes of type H and faces a wage distribution $F_H(.)$ with support $[\underline{w}_H, \overline{w}_H]$. We assume that $F_H(.)$ first order stochastically dominates $F_L(.)$, so one can think of male types as representing productivities. Immediately after the schooling decision, all men (H and L) become active in both the labour market and the marriage market. In the labour market, they look for wage offers using costless random sequential search, and job opportunities arrive at rate λ_0. If employed at wage w, a man receives the flow payoff w, and we assume there is no on-the-job search. While active in the labour market, single men also use costless sequential search to look for partners in the frictional marriage market. Marriage requires mutual acceptance, and we assume that divorce is not possible. For a man, marriage confers a flow payoff y which captures the non-economic utility of the partnership. Overall therefore, a married man employed at wage w has a flow payoff $w + y$.

Women enter the economy single, and let $x > 0$ denote the flow payoff of a single woman. This parameter is crucial for our investigation, as it captures a woman's option outside marriage. Here, we interpret this as her career opportunities, so an increase in x would mean higher labour market returns, possibly due to higher schooling. Single women look for males using costless sequential search, but (as we will show) they are not interested in marrying unemployed men. Hence, for them the relevant wage distribution is that of wages earned by single type i men, denoted by $G_i(.)$. Once a marriage partnership is formed, there is no intra-marital bargaining over the surplus, and a married woman simply enjoys a flow utility equal to their partner's wage.

Since for both sexes utilities are monotonic in wages, with sequential search the optimal strategies for both women and men are characterised by the reservation property. Let R_i denote the reservation wage of unemployed type i men in the labour market. Similarly, let T_i denote the reservation wage of women in the marriage market, meaning an employed man of type i is marriageable only if his wage is no lower than T_i.

Everyone (irrespective of employment- and marital status) leaves the economy at rate δ. Let Γ denote the exogenous flow (measure) of new (unemployed and single) men who enter the economy at every instance, and let N_i denote the number of marriageable employed single men of type i. Similarly, let n denote the measure of single women; it is exogenous as we assume that a new single woman comes into the market every time a single woman gets married or exits the economy. Denote by λ_w^i the rate at which a single woman meets an eligible bachelor, and let λ_m denote the rate at which single men meet single women. We assume a quadratic matching function with parameter λ that measures the efficiency of the matching process. Then, we have $\lambda_w^i = \frac{\lambda(N_H+N_L)n}{n}\frac{N_i}{(N_H+N_L)} = \lambda N_i$, and $\lambda_m = \frac{\lambda(N_H+N_L)n}{(N_H+N_L)} = \lambda n$, where both N_i and λ_w^i are of course endogenous. Crucially, let τ_H denote the equilibrium (endogenous) proportion of male entrants who decided to invest in schooling. The main questions we ask are: (i) how does the steady state equilibrium fraction of educated men change with x (comparative statics), and (ii) when is this mixed equilibrium stable?

3 Steady state and optimal search

3.1 Unemployed men, marriageable men and wages

Let u_i denote the number of unemployed men ot type i. In steady state we require $\Gamma\tau_H = u_H[\delta + \lambda_0(1 - F_i(R_i))]$. That is, the inflow of unemployed men who have chosen to invest in schooling needs to equal the outflow of these educated unemployed, either into employment (at an acceptable wage) or full exit.

Consequently, in steady state we have:

$$u_H = \frac{\tau_H \Gamma}{\delta + \lambda_0 \left[1 - F_H(R_H)\right]}$$

and

$$u_L = \frac{(1-\tau_H)\Gamma}{\delta + \lambda_0 \left[1 - F_L(R_L)\right]}.$$

In order to obtain the number of single marriageable men of type i, we simply need:

$$u_i \lambda_0 [1 - F_i(T_i)] = N_i(\lambda n + \delta).$$

Using u_i as above, we get:

$$N_i = \frac{\tau_i \Gamma}{\delta + \lambda [1 - F_i(R_i)]} \frac{\lambda_0 [1 - F_i(T_i)]}{\lambda n + \delta}.$$

Note the role of u_i in the determination of N_i. We will show that when the marriage market affects unemployed men's search behaviour, there are two possible outcomes:

When $R_i < T_i$ the number of marriageable men increases with R_i. Given the exogenous wage distributions, if the reservation wages R_i increase, men of type i leave unemployment at a lower rate, so the steady state u_i increases. Then, since the rate at which men accept marriageable wages remains unchanged, this results in an increase in N_i. Furthermore, N_i increases when T_i decreases.

In turn, when R_i is optimally set equal to T_i an increase in R_i results in a decrease in N_i.

Finally, the distribution of wages earned by marriageable men of type i is given by the steady state condition:

$$u_i \lambda_0 [F_i(w) - F_i(T_i)] = G_i(w) N_i(\lambda n + \delta).$$

The number of marriageable men of type i with wages no higher than w is $G_i(w)N_i$. They leave this stock if they get married or exit the economy altogether. The left-hand side captures the flow of unemployed men of type i who find and accept a job with a wage that confers marriageability but is no higher than w.

Then, using the solution for N_i above, we have:

$$G_i(w) = \frac{F_i(w) - F_i(T_i)}{1 - F_i(T_i)}.$$

3.2 Optimal search: women

In this section, we derive the female reservation wages T_i. To do this, we first establish that women refuse to marry unemployed men of type i if the female reservation wage is high enough.[4] Consider a married and employed man of type i. Without the possibility of either on-the-job search or divorce, standard considerations give the discounted expected lifetime utility of such a man:

$$V_i^M(w) = \frac{w+y}{r+\delta}.$$

Although the above utility is clearly independent of type (education), in the interest of clarity we will continue to use the subscripts whenever we refer to this value of employment.

Now consider a married but unemployed man of type i. As he is no longer active in the marriage market, his reservation wage is simply the standard pure labour market one:

$$\underline{R}_i = \frac{\lambda_0}{r+\delta} \int_{R_i^M}^{\overline{w}_i} [1 - F_i(w)]\, dw \tag{1}$$

Note that $\underline{R}_H > \underline{R}_L$ because type H men have better job prospects in the labour market. Furthermore, as we will show later, $\underline{R}_i$ is in fact the lowest reservation wage for each type.[5]

In principle, women could of course marry unemployed men as well. Let us therefore examine the situation of a woman who is married to a jobless type i man. Her value function W_i^U is given by:

$$(r+\delta)W_i^U = \lambda_0 \int_{\underline{R}_i}^{\overline{w}_i} \left[W_i^M(w) - W_i^U\right] dF_i(w),$$

[4]This particular bit of analysis mirrors to some extent the one carried out in Bonilla et al. (2019), with the crucial difference that in this paper male types (here productivity/education) are endogenous.

[5]In Bonilla et al. (2019) we also have $\underline{R}_i$ same for all types, but the actual meaning of "types" is entirely different there.

where $W_i^M(w) = w/(r+\delta)$ is the discounted lifetime utility of being married to a type i employed man who earns wage w. The above equation incorporates the fact that a married type i unemployed man has reservation wage $\underline{R}_i$.

For T_i to be a female reservation wage, it needs to satisfy the condition $W_i^U = T_i/(r+\delta)$. Given $W_i^M(w)$, we have:

$$T_i = \frac{\lambda_0}{r+\delta} \int_{\underline{R}_i}^{\overline{w}_i} [w - T_i] \, dF_i(w),$$

and the unique solution to this is $T_i = \underline{R}_i$. Now, if a woman's value of being single (denoted from now on by W^S) increases, her reservation wage also increases. In contrast, W_i^U is independent of T_i. Hence, $W^S \lesseqgtr W_i^U$ if and only if $T_i \lesseqgtr \underline{R}_i$, and therefore we can conclude that if $T_i > \underline{R}_i$, women will reject marriage to a type i unemployed man. Throughout, we work under the assumption that this is indeed the case.[6]

Next, we turn to the actual derivation of women's reservation wages T_i, emphasising that women cannot direct their search efforts and therefore contact with an H or an L man is completely random. Importantly, $W_H^M(w) = W_L^M(w) = w/(r+\delta)$: as this man is already employed, his type is actually irrelevant for the woman. Using the definition of a reservation value we have $W^S = W_H^M(T_H) = W_L^M(T_L)$, which implies $T_H = T_L$. Therefore, from now we drop the subscripts and use $T(= T_H = T_L)$ instead.

Recall that W^S denotes the value of being single for a woman. Standard derivations lead to the Bellman equation:

$$\begin{aligned}(r+\delta)W^S &= x + \lambda N_H \int_T^{\overline{w}_H} \left[W_H^M(w) - W^S\right] dG_H(w) + \\ &\quad + \lambda N_L \int_T^{\overline{w}_L} \left[W_L^M(w) - W^S\right] dG_L(w).\end{aligned}$$

[6] By doing so, we essentially eliminate the uninteresting equilibrium where the marriage market does not affect men's job search.

Making use of the solutions for N_i and $G_i(w)$ obtained above, this becomes:

$$(r+\delta)W^S = x + \frac{\lambda\tau_H\Gamma\lambda_0}{[\delta+\lambda(1-F_H(R_H))](\lambda n+\delta)}\int_T^{\overline{w}_H}\left[W_H^M(w)-W^S\right]dF_H(w) + \\ +\lambda\tau_L\frac{\lambda(1-\tau_H)\Gamma\lambda_0}{[\delta+\lambda(1-F_L(R_L))](\lambda n+\delta)}\int_T^{\overline{w}_L}\left[W_L^M(w)-W^S\right]dF_L(w).$$

Finally, using $W^S = T/(r+\delta)$ and applying standard integration by parts, we obtain:

$$T = x + \frac{\lambda\tau_H\Gamma\lambda_0}{[\delta+\lambda(1-F_H(R_H))](\lambda n+\delta)}\int_T^{\bar{w}_H}[1-F_H(w)]dw + \\ +\frac{\lambda(1-\tau_H)\Gamma\lambda_0}{(\delta+\lambda[1-F_L(R_L)])(\lambda n+\delta)}\int_T^{\bar{w}_L}[1-F_L(w)]dw. \qquad (2)$$

At this point, we would like to highlight three observations that are important for what follows:

1. Clearly, $\partial T/\partial x > 0$: as expected, women raise their reservation wage in the marriage market if their instantaneous utility from staying single increases.
2. $\partial T/\partial \tau_H > 0$: intuitively, a ceteris paribus increase in the fraction of educated men with better job prospects makes women pickier, as their own marriage market prospects have improved.
3. $\partial T/\partial R_i > 0$: again, ceteris paribus, a higher reservation wage of a given type i man increases the number of marriageable men (see the discussion around N_i above), so women can afford to become choosier.

3.3 Optimal search: men

We are interested in equilibria in which the marriage market does affect all men's decisions in the labour market. Nevertheless, it is instructive to consider the optimal job search behaviour of men under all possible circumstances. To that end, first recall that in any scenario where the marriage market does *not* influence labour market decisions, the male reservation wage is given by $\underline{R}_i$ obtained above.

When the marriage market *does* have an effect (through T) on male strategies, single unemployed men undertake a so-called *constrained* search, knowing that by accepting a particular wage (for life), they either become marriageable or loose the prospect of marriage forever. As a consequence, a man of type i searches from the wage offer distribution $F_i(w)$ and, for any given female reservation wage T which makes him acceptable for marriage, he uses a reservation wage *function* $R_i(T)$.

In what follows, we fully characterise the function $R_i(T)$. Although the derivation of the reservation wage function will be the same for both types[7], the respective actual reservation wage functions will be different across types, essentially due to the fact that men with different schooling choices face different wage distributions. The main insight is that this function is non-monotonic in the female reservation wage, and has a unique maximum, attained at $T = \widehat{T}_i$, where the latter is defined by:

$$\widehat{T}_i = \frac{\lambda_0}{r+\delta}\left[\int_{\widehat{T}_i}^{\overline{w}_i} [1 - F_i(w)]\, dw + \frac{\lambda n \left[1 - F_i(\widehat{T}_i)\right]}{r + \delta + \lambda n} y\right]. \tag{3}$$

Clearly, for $y > 0$ and $F_i(\widehat{T}_i) < 1$, we have $\widehat{T}_i > \underline{R}_i$.

The formal reasoning is as follows: Overall, a man (of either type) can be in one of three states: unemployed and single, employed at wage w and single (S), or employed at wage w and married (M). Denote a type i man's value of being unemployed by U_i, and let $V_i^S(w)$ describe the value of being single and earning a wage w. Standard derivations lead to the Bellman equation for a type i unemployed man:

$$(r+\delta)U_i = \lambda_0 \int_{\underline{w}_i}^{\overline{w}_i} \max\left[V_i^S(w) - U_i, 0\right] dF_i(w).$$

[7] For a much more detailed exposition of this, please consult Bonilla et al. (2019).

Anticipating that $V_i^S(w)$ is not a continuous function (see below), we can define:

$$R_i(T) = \min\left\{w : V_i^S(w) \geq U_i\right\}.$$

Since there is no divorce, the value of being married and earning a wage w is $V_i^M(w) = (w+y)/(r+\delta)$. Hence, for any T, we have:

$$V^S(w) = \left\{ \begin{array}{ll} \frac{w}{r+\delta} & \text{if} \quad w < T \\ \frac{w}{r+\delta} + \frac{\lambda n}{(r+\delta+\lambda n)(r+\delta)} y & \text{if} \quad w \geq T \end{array} \right\}.$$

Please note that when $\lambda n = 0$ (i.e. no marriage market), we have $V_i^S(w) = w/(r+\delta)$ for all w. Then, from $U_i = V_i^S(R_i)$, standard manipulation yields $R_i = \underline{R}_i$. As stated before, this is also the reservation wage that would be chosen by a hypothetical unemployed married man since, without divorce, this man is no longer involved in the marriage market.

The Proposition below presents the full characterisation of the male reservation wage function.

Proposition 1 *The reservation wage function $R_i(T)$ is continuous, and:*
(a) $R_i = \underline{R}_i$ for $T \leq \underline{R}_i$ and $T > \overline{w}_i$;
(b) $R_i = T$ for $T \in (\underline{R}_i, \widehat{T}_i]$;
(c) $R_i < T$ and decreasing for $T \in (\widehat{T}_i, \overline{w}_i]$.
Furthermore, $\widehat{T}_H > \widehat{T}_L$ and $\underline{R}_H > \underline{R}_L$.

Proof. First, consider $\overline{w}_i > T \geq \widehat{T}_i$. Assume for a moment that $R_i(T) \leq T$. Then, using $V_i^S(R_i) = R_i/(r+\delta) = U_i$, $R_i(T)$ is given by:

$$R_i(T) = \frac{\lambda_0}{r+\delta} \int_{R_i}^{\overline{w}_i} [1 - F_i(w)]\, dw + \frac{\lambda_0 \lambda n\,[1 - F_i(T)]}{(r+\delta)(r+\lambda n+\delta)} y.$$

From the above, $R_i(\widehat{T}_i) = \widehat{T}_i$, where $\widehat{T}_i$ as defined in (3). Call this reservation wage $\widehat{R}_i$. Also, from the above, when $T \geq \overline{w}_i$ we have $R_i(T) = \underline{R}_i$, since then $F_i(T) = 1$. It is easy to show that $\widehat{T}_i < \overline{w}_i$, and $R_i(T)$ is decreasing in T. Hence, $R_i(T) < T$ iff $T > \widehat{T}_i$. For $T \leq \widehat{T}_i$, the reservation function derived above does not survive as an optimal strategy. Also, unemployed men are indeed not marriageable since a married unemployed would choose $\underline{R}_i (< T)$.

Now consider $\underline{R}_i < T < \widehat{T}_i$. Unemployed men are still not marriageable. The value of being a single unemployed with a reservation wage $R_i > T$ is given by:

$$(r+\delta)U_i = \frac{\lambda_0}{r+\delta}\int_{R_i}^{\overline{w}_i}[1-F_i(w)]dw + \frac{\lambda_0\lambda n}{(r+\delta)(r+\delta+\lambda n)}y.$$

On the other hand, when choosing T as the reservation wage, this value is given by:

$$(r+\delta)U_i = \frac{\lambda_0}{r+\delta}\int_{T}^{\overline{w}_i}[1-F_i(w)]dw + \frac{\lambda_0\lambda n}{(r+\delta)(r+\delta+\lambda n)}y.$$

For $R_i > T$, the latter is higher than the former. Intuitively, the only reason to increase R_i above $\underline{R}_i$ would be in order to become marriageable. But $R_i = T$ is already enough for that.

Next, consider $T \leq \underline{R}_i$. If men believe they are marriageable irrespective of their employment status, they choose $R_i = \underline{R}_i$ both when single and married. This is because $V_i^S(w) = \frac{w}{r+\delta} + \frac{\lambda n}{(r+\delta+\lambda n)(r+\delta)}y$ for all $w \geq \underline{R}_i$. For $T < \underline{R}_i$, they are indeed always marriageable.

Now consider the case with $T > \overline{w}_i$. We have $1 - F_i(T) = 0$, and therefore a man of type i can never get married (as the highest available wage is $\overline{w}_i$). Men optimally set $R_i(T) = \underline{R}_i$.

Finally, $\widehat{T}_H > \widehat{T}_L$ and $\underline{R}_H > \underline{R}_L$ follow from the fact that $F_H(w)$ first order stochastically dominates $F_L(w)$. ■

In essence, when the marriage market affects men's job search strategy, unemployed males can react in two ways. For relatively low values of female reservation wages, they choose to hold out for such wages and commit to $R_i = T$. For relatively high female reservation wages (higher than $\widehat{T}_i$) men gradually give up on trying to match T, so they only get married if they are lucky and land a high enough wage.

Two further observations follow from the above. First, men's value of unemployment U_i is not directly affected by x, so $\partial R_i(T)/\partial x = 0$ as women's flow utility of being single does not affect the male reservation wage functions. However, note that x will of course affect the equilibrium male reservation wages, through its direct effect on the female reservation wage T. Second, the value of unemployment is not directly affected by τ_H either, so $\partial R_i/\partial \tau_H = 0$.

4 Equilibrium

In this section we investigate the existence and properties of a market equilibrium by first looking at the partial search equilibrium in the joint frictional markets, and then (using backward induction), pinning down the steady state fraction of educated men that is consistent with optimal schooling investment choices. Intuitively, as they face a binary decision, men will choose to invest in schooling as long as the returns from education - as captured here by the difference in the values of educated and uneducated single unemployed men, is higher than the cost of schooling. A mixed equilibrium with only a fraction of educated men therefore requires $\Delta U(\equiv U_H - U_L) = c$, meaning all males are indifferent between investing or not in schooling.

The *exogenous* parameter x plays a key role in the determination of the partial equilibrium in the labour and marriage markets, where agents also take τ_H as given. Crucially however, τ_H is the *only endogenous* variable that can adjust to ensure the equality of returns to education and cost of schooling.

The central results of our paper concern the nature of the interaction between these two variables. To get a flavour of the argument, consider a change in the flow utility of single women, x. This generates a change in $T(R_H, R_L)$ *only* - recall that the value of unemployment and reservation wages of men is not directly affected. However, the shift in $T(R_H, R_L)$ itself has an immediate impact on U_H and U_L, due to the change in the proportion of marriageable wages (different across male types). This leads to an adjustment in male reservation wages and with that, also a change in the returns to education. With comparative statics in mind, recall that, just like for x, we have $\partial U_i / \partial \tau_H = 0$ so τ_H affects $T(R_H, R_L)$ only. Therefore, τ_H is indeed the only endogenous variable that can adjust in order to restore ΔU to its equilibrium level.

We also address the question of stability of the market equilibrium. To that end, given the change in returns to education due to a change in x, we examine the conditions under which men's optimal educational decision leads to a change in τ_H in the direction required to restore the original equilibrium level of ΔU.

4.1 Partial search equilibrium

First, note that the number of steady-state educated single unemployed men (u_H) is essentially determined by the proportion of men who decide to invest in schooling, τ_H. Taking these two measures *as given*, a search equilibrium for the inter-linked frictional markets is the triplet $\{R_L^*, R_H^*, T^*\}$ together with steady state conditions, such that male reservation wages satisfy Proposition 1 and the female reservation wage satisfies (2). There are three types of potential equilibria: Type 1, characterised by $R_i^* < T^*$ $(i = L, H)$, $R_L^* < R_H^*$ and $\partial R_i/\partial T < 0$; Type 2, characterised by $R_H^* = T^*$, $R_L^* < T^*$ and $\partial R_L/\partial T < 0$; and Type 3, with $R_i^* = T^*$.

Proposition 2 *A partial search equilibrium exists and it is unique. In any such equilibrium $\partial T^*/\partial x > 0$ and $\partial T^*/\partial \tau_H > 0$.*

Proof. To show existence, note that $R_i(T)$ is continuous and non-monotonic in T, while $T(R_i)$ is continuous and increasing in R_i.

The proof of the second statement is by contradiction. Consider a potential Type 1 equilibrium. Let x increase, and assume a resulting new equilibrium with *lower* T^*. Since $\partial R_i/\partial T < 0$ while the reservation wage of a particular male type is not directly affected by the reservation wage of the other type, this would necessarily involve higher R_i^*. From (2), a higher x, together with higher R_i^* unambiguously results in a higher female reservation wage. Consider next a potential Type 3 equilibrium, and increase x. Imposing $R_i^* = T^*$ in (2) we have $\partial T/\partial x > 0$, and therefore a new equilibrium with a lower T^* would be a contradiction. Finally, consider a potential Type 2 equilibrium, and increase x. Given that $\partial R_L/\partial T < 0$ and R_H^* is optimally set equal to T^*, the above two arguments once again imply that a new equilibrium with a lower T^* would be a contradiction.

The reasoning which proves that $\partial T^*/\partial x > 0$ also implies that the female reservation wage function implicitly given in (2) crosses the 45 degree line from below, and uniqueness follows. Finally, an increase in τ_H shifts the female reaction function in (2) to the right. ■

Although hidden, the role played by N_i in the results above is worth stressing. For the Type 1 equilibrium, an increase in R_i leads to an increase in N_i, which (ceteris paribus) makes women pickier. This, coupled with an increse in x must result in a higher female reservation wage. For an equilibrium of Type 2 that has $R_i^* = T^*$, a lower equilibrium female reservation wage would mean a higher N_i. But the combined effect of both a higher N_i and a higher x is that single women become pickier.

Panel (a) in Figure 1 captures a Type 1 partial search equilibrium for the joint labour and marriage markets, with the female reservation wage graphed against the reservation wage of educated men. Panel (b) captures the *same* partial search equilibrium, but this time with the female reservation wage graphed against the reservation wage of uneducated men.

Figure 1

Note that in Panel (b) the female reservation wage (graphed against R_L) is positioned more to the right compared with the case depicted in Panel (a), where it is graphed against R_H. This configuration is always true for $R_H > R_L$.

Finally, we are now also in the position to describe the range of the parameter x for which different types of equilibria obtain. To this end, define x_1, x_2, x_3, and x_4 such that $T^*(x_1) = \underline{R}_H$, $T^*(x_2) = \widehat{T}_L$, $T^*(x_3) = \widehat{T}_H$, and $T^*(x_3) = \overline{w}_L$. Then, for $x \in (x_3, x_4]$ a Type 1 equilibrium exists; for $x \in (x_2, x_3]$ a Type 2 equilibirum exists, and a Type 3 equilibrium exists for $x \in (x_1, x_2]$.

4.2 Market equilibrium with schooling

Our focus is on a mixed market equilibrium characterised by (i) Type 1 partial search equilibrium in the joint frictional markets and (ii) a fraction of men who choose schooling. Men will choose to invest in schooling as long as $\Delta U \equiv U_H - U_L > c$, and hence an interior equilibrium requires that $\Delta U = c$, so all men are indifferent between paying or not for education. Because in a Type 1 partial search equilibrium $U_i = R_i/(r + \delta)$ the condition for such a mixed market equilibrium amounts to:

$$\Delta R^* (\equiv R_H^* - R_L^*) = \frac{c}{r + \delta}.$$

Clearly, this equality pins down the equilibrium value of returns to education ΔR^*. Since $\partial R_i(T)/\partial T < 0$ for $T > \widehat{T}_H$, while $\partial R_L(T)/\partial R_H =$

$\partial R_H(T)/\partial R_L = 0$, the above equation also pins down the equilibrium value(s) of T^*, and with it the associated equilibrium values of R_H^* and R_L^*.[8].

In addition, recall that although both x and τ_H affect $T(R_i)$ directly, with $\partial T(R_i)/\partial x > 0$ and $\partial T(R_i)/\partial \tau_H > 0$, they have no direct effect on the $R_i(T)$ themselves, so $\partial R_i(T)/\partial x = \partial R_i(T)/\partial \tau_H = 0$. In equilibrium, a change in x affects the triplet $\{R_H^*, R_L^*, T^*\}$ through its direct effect on $T(R_i)$ *only*. Furthermore, although the proportion of men who choose to invest in schooling is endogenous in the overall market equilibrium, it acts as a *parameter* in the determination of T^* in the partial search equilibrium. As a consequence, our mixed equilibrium condition boils down to:

$$\Delta R^*(T^*(x, \tau_H) = \frac{c}{r + \delta} \tag{4}$$

Crucially, it is the proportion of men who choose to get educated (τ_H) that ensures the above equilibrium condition is satisfied. Denoting by τ_H^* the equilibrium value of this endogenous variable, we are particularly interested in the effect of an increase in single women's flow utility on the fraction of men who invest in schooling, τ_H^*. To carry out this comparative statics exercise, assume that condition (4) is satisfied, and consider an increase in x. Ceteris paribus, this positive shock increases the female reservation wage (see Proposition 2), so women become pickier in the marriage market. In a Type 1 partial search equilibrium, this leads to a decrease in the reservation wages of both educated and uneducated men (R_i^*). But then the equilibrium condition (4) does not hold anymore, as returns to education are affected. Only a decrease in the fraction of men who invest in education can reduce T^* (again, see Proposition 2), and thereby re-adjust the labour market returns ΔR^*, back to their original equilibrium level. Our main result below formalises this argument:

Theorem 1 *Consider a mixed market equilibrium (MME) characterised by:*
(i) $x \in (x_3, x_4]$
(ii) R_i^* *as in Proposition* 1(*c*)
(iii) T^* *given by* (2)
(iv) $\tau_H^* \in (0, 1)$ *solves* (4)
In such an equilibrium, $\partial \tau_H^*/\partial x < 0$.

[8] If ΔR is monotonic in T (and this of course depends on the distribution functions F_i), the condition in fact pins down a *unique* triplet $\{T^*, R_H^*, R_L^*\}$.

Proof. A MME satisfies (4), and requires that $d\Delta R^*(T^*(x,\tau_H)) = 0$ for any joint change in key parameters of the partial search equilibrium. Total differentiation of (4) yields $d\Delta R^*(T^*(x,\tau_H)) = \frac{\partial \Delta R^*}{\partial T^*}\frac{\partial T^*}{\partial x}dx + \frac{\partial \Delta R^*}{\partial T^*}\frac{\partial T^*}{\partial \tau_H}d\tau_H$. Assume for a moment that $\partial\Delta R^*/\partial T^* < 0$. Since $\partial T^*/\partial x > 0$ (see Proposition 2), an increase in x leads to a higher T^*, and this implies $\frac{\partial \Delta R^*}{\partial T^*}\frac{\partial T^*}{\partial x}dx < 0$. Hence, $d\Delta R^*(T^*(x,\tau_H)) = 0$ only if $\frac{\partial \Delta R^*}{\partial T^*}\frac{\partial T^*}{\partial \tau_H}d\tau_H > 0$. As $\partial T^*/\partial\tau_H > 0$ (also see Proposition 2), this in turn requires a lower τ_H. Next, assume for a moment that $\partial\Delta R^*/\partial T^* > 0$. Since $\partial T^*/\partial x > 0$, an increase in x leads to a higher T^*, and this implies $\frac{\partial \Delta R^*}{\partial T^*}\frac{\partial T^*}{\partial x}dx > 0$. Hence, $d\Delta R^*(T^*(x,\tau_H)) = 0$ only if $\frac{\partial \Delta R^*}{\partial T^*}\frac{\partial T^*}{\partial \tau_H}d\tau_H < 0$. As $\partial T^*/\partial\tau_H > 0$, this again requires a lower τ_H. ■

The result that the effect of an increase in x on the equilibrium proportion of men who choose education is *always* negative deserves a closer look. This strong result follows because, while a change in x alters the actual returns to education through its effect on T, it does not alter the value of these returns that is consistent with the equilibrium, and hence neither does it affect the value of T that is consistent with the equilibrium. This, together with the fact that $\partial T/\partial x > 0$ and $\partial T/\partial\tau_H > 0$, delivers our main result.

What about the stability of this mixed market equilibrium? The way men change their education choice after a change in x that affects the female reservation wage is crucial since it determines the direction of adjustment of τ_H. In turn, all this is essentially determined by the effect on the returns to education ΔR. In particular, first note that an increase in T has a negative effect on all men: it leads to a decrease in the proportion of *offered* marriageable wages that is also *different* across male types. Indeed, we have $\partial\Delta R/\partial T < 0$ if the negative effect of an increase in T is stronger for H than for L type men, that is:

$$\frac{\partial F_H(T)}{\partial T} > \frac{r+\delta+\lambda_0(1-F_L(R_L)]}{r+\delta+\lambda_0(1-F_H(R_H)]}\frac{\partial F_L(T)}{\partial T} \tag{5}$$

Intuitively, the returns to education diminish as the female reservation wage increases if the increase in the proportion of unmarriageable wages in the distribution $F_H(.)$ faced by educated men is high enough relative to that in the distribution faced by uneducated men $F_L(.)$, where "high enough" takes into account the fact that the female reservation wage affects employment probabilities through its effect on male reservation wages. Proposition 3 below formalises the above argument in terms of the direction of adjustment for τ_H.

Proposition 3 *Consider a mixed market equilibrium (MME). If inequality* (5) *holds, an increase in x leads to a decrease in τ_H. If the inequality* (5) *holds in the opposite direction, an increase in x leads to an increase in τ_H.*

Proof. In the MME we have $\Delta R^*(T^*(x, \tau_H)) = c/(r+\delta)$. An increase in x leads to an increase in T^*. When (5) holds we have $\partial \Delta R/\partial T < 0$, and therefore now $\Delta R(T) < c/(r+\delta)$, so τ_H adjusts downwards (reversing T to T^*) until either the original equilibrium is restored or a corner solution emerges, with $\tau_H = 0$. When (5) holds in the opposite direction, $\partial \Delta R/\partial T > 0$, and hence $\Delta R(T) > c/(r+\delta)$, so τ_H adjusts upwards, thus increasing T even further. If ΔR is monotonic, this process continues until $\tau_H = 1$; if ΔR is not monotonic, it is possible that the equilibrium condition $\Delta R^*(T^*(x, \tau_H)) = c/(r+\delta)$ is met for a *different* T^*. ■

Given the above stability condition for our mixed market equilibrium, we can spell out in detail and interpret the chain of reactions that follow a positive shock in women's options outside marriage. In our model, such a shock is meant to capture changes in female labour market returns that are either partly or entirely due to enhanced schooling investment on their part. The immediate effect of any such change is that women become pickier in the marriage market. In turn, the resulting increase in the female reservation wage has an adverse effect on the men active in the marriage market, as it leads to a decrease in the proportion of marriageable wages, for both educated and uneducated single men. Unemployed men adjust their labour market strategy, with all males reducing their reservation wages. If the increase in x (through the increase in T) harms the *marriage* market prospects of educated men relatively more than those of uneducated men, so the former reduce their reservation wage more than the latter, the respective changes in male reservation wages adds up to a decrease in the returns to schooling. But then the equilibrium condition in (4) does not hold anymore, and as we have discussed, the fraction of men who undertake investment in education decreases. At this point, the counterveiling effect kicks in. With fewer educated men around, women become less picky, and the associated decrease in the female reservation wage continues until it reaches its old level. Only then will men be once again indifferent between acquiring or not education. Interestingly, the logic of the above transition mechanism suggests that if women experience a positive shock in the labour market (higher returns, possibly due to increased educational attainment), this is entirely offset by a negative effect on the marriage market, where their prospects suffer as the pool of educated eligible men shrinks.

Finally, what about the other two types of possible market equilibria (characterised by Type 2 and Type 3 partial search equilibria)? In terms of existence and uniqueness, the exact same arguments apply, suitably adjusted by substituting U_i for $R_i/(r+\delta)$. As far as the stability of these alternative equilibria is concerned, it can be shown that $\partial F_H(T)/\partial T > \partial F_L(T)/\partial T$ is in fact sufficient for the stability market equilibria with either Type 2 or Type 3 partial search equilibrium.

References

[1] Becker, G., *A Treatise on the Family*, 3rd edition (Cambridge, MA: Harvard University Press, 1991).

[2] Blau F., L. Kahn, and L. Waldfogel, "Understanding Young Women's Marriage Decisions: the Role of Labor and Marriage Market Conditions," *Industrial & Labor Relations Review* 53 (2000), 624-47.

[3] Bonilla, R., and F. Kiraly, "Marriage Wage Premium in a Search Equilibirum," *Labour Economics* 24 (2013), 107–15.

[4] Bonilla, R., F. Kiraly, and J. Wildman, "Beauty Premium and Marriage Premium in Search Equilibrium: Theory and Empirical Test," *International Economic Review* (2019).

[5] Browning, M., P-A. Chiappori, and Y. Weiss, *Economics of the Family*, (Cambridge: Cambridge University Press, 2014).

[6] Chiappori, P-A, M. Iyigun, and Y. Weiss, "Investment in Schooling and the Marriage Market," *American Economic Review*, 99 (2009), 1689–1713.

[7] Flinn, C., and J. Mullins, "Labour Market Search and Schooling Investment," *International Economic Review*, 56 (2015), 359-98.

[8] Goldin, C., L.F. Katz, and I. Kuziemko, "The Homecoming of American College Women: The Reversal of the College Gender Gap," *Journal of Economic Perspectives*, 20 (2006), 133-56.

[9] Gould, E.D., and M.D. Paserman, "Waiting for Mr. Right: Rising Wage Inequality and Declining Marriage Rates," *Journal of Urban Economics*, 53 (2003) 257-81. (2003).

[10] Oppenheimer, V.K., "A Theory of Marriage Timing," *The American Journal of Sociology*, 94 (1988), 563–91.

[11] Oppenheimer, V.K., and V. Lew, "American Marriage Formation in the 1980s: How Important Was Women's Economic Independence?," in K.O. Mason, and A-M. Jensen, eds., *Gender and Family Change in Industrialized Countries*. (Oxford: Clarendon Press, 1995).

[12] Pissarides, C., *Equilibrium Unemployment Theory*, 2nd edition (Cambridge, MA: MIT Press, 2000).

Figure 1: Partial search equilibrium