Going up and down by the lift to learn Linear Algebra

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Abstract

We have developed several engineering problems and situations according to the different mathematical levels in engineering degrees. In this paper we show how students could learn the calculation of eigenvalues and eigenvectors solving a mechanical problem of vibrations. From a one degree of freedom system and its physical and mechanical knowledge, students are able to acquire the mathematical competencies and get the values of frequencies and vibrations modes. We present in this study the results from linear algebra course from different years, using different methodologies and also different assessment methods.

Introduction

Calculus, Linear Algebra, Numerical Methods, and in general mathematics, could be considered as tools in the training of engineering students. In recent years, teaching and learning methodologies have changed considerably. We moved away from master classes with separate hours of theory and problems, to different attempts to apply mathematics to engineering courses, which motivates students and make them acquire the mathematical competencies.

The aim of this paper is to present a proposal to make students acquire the 8 competencies proposed in the Framework document from the mathematics working group (Alpers et al., 2013): Thinking mathematically, reasoning mathematically, posing and solving mathematical problems, modelling mathematically, representing mathematical entities, handling mathematical symbols and formalisms, communicating in, with, and about Mathematics, and making use of aids and tools for mathematical activities.

The Bologna process has made university teachers to promote a change in the educational paradigm. As we teach to science and engineering students, we must keep in mind that these students are different from the ones in mathematics degrees. The way of teaching and learning have became a new challenge as mathematics represents a tool and not a goal in itself.

One of the changes that came with the degrees modification is the use of technology for educational purposes. In recent years, apart from using a moodle-based learning platform, we also use different mathematical software, such as Mathematica or Matlab. Moreover, during some courses we propose the use of Socrative or Kahoot as gamification and assessment tools for quick answers from students (Bullón et al., 2018).

In Figure 1 we show our current situation concerning industrial engineering students at the University of Salamanca, from electricity, automatics, and mechanics. There is a continuous decrease of the success rate. In fact, in the 2016-17 academic course, from 74 students (22 from electricity, 27 from electronics, and 25 from mechanics) only 11 students (5 from electricity, 4 from electronics, and 2 from mechanics) were promoted to the next grade. These quantities are not a good quality indicator. This situation leads as to propose a change in the teaching and learning methodology.

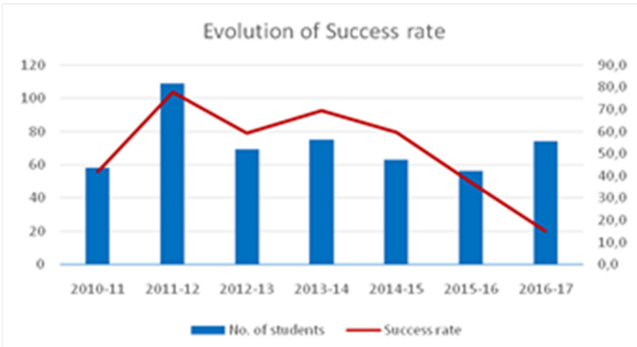

Figure 1. Results from Mathematics (linear algebra) subject from industrial engineering studies at the University of Salamanca.

The linear algebra subject, for engineering students, at the University of Salamanca, includes 5 blocks of contents: (1) Systems of linear equations, (2) vector spaces, (3) linear applications and associated matrix, (4) Euclidean space, and (5) diagonalization. This represents a basic subject in the first semester of the bachelor's degrees.

Methodology

We started with the proposal of using the mobile phone as a pendulum together with a computer algebra system. Once students collect the data from their devices, they were able to calculate the frequency of a pendulum or a spring and this leads them to study the vibration of a system with one, two or more degrees of freedom. Mobile-aided learning and computer-based learning in general help students to be motivated. The use of devices awakens their curiosity and captures their attention. We have tested this with students from the "master degree in teachers from compulsory secondary education and high school, vocational training and language teaching" at the same University. All students (around 10) were very motivated with the use of the mobile phone for mathematics and physics classes. The results were very positive.

The proposal now is to develop a competencies-based course to improve the final marks in the case of students from the first year of bachelor degrees. This will indicate that students acquire the competencies, which improve their engineering education, and make them be prepared for their future careers.

The activity that we propose is the use of the mobile phone to present some algebraic problems to make more understandable the subject's contents and, at the same time, to make students acquire the competencies.

The phyphox App, available for Android and iOS (http://phyphox.org/), includes several physical experiments to work with students: Elastic collision, acceleration (without and without g), acceleration with g, audio amplitude, audio autocorrelation, centrifugal acceleration, Doppler effect, elevator, free fall, etc. All of them include videos and working activities to do it easy to use.

The first activity that we propose for the Linear Algebra curriculum is the use of the "elevator" from phyphox App to make students understand the concept of a vector, as an element with a direction and a value (module). When the mobile is use to measure the acceleration of the elevator students get in their mobile screens the Figure 2.

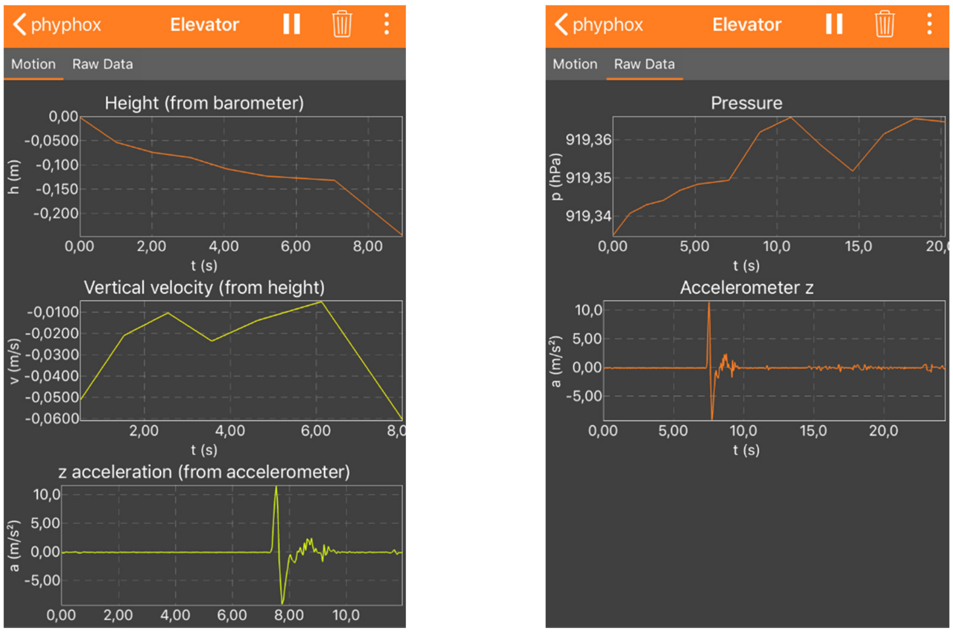

Figure 2. Phyphox working with the elevator experiment.

Once the students understand the concept of vectors and the meaning of different magnitudes, it is possible to work with vector spaces (collection of objects called vectors) and subspaces, as these keep the properties of the vector spaces.

The methodology that has been used in this study is similar to the approaches that are made in the PISA tests, that is, to develop Mathematical competencies from problematic situations, in our case from engineering problems, motivating students to reflect and propose solutions in the context of industrial engineering. In this context we speak about mathematizing, meaning the fact of applying methods and mathematical approach to different experiments.

The experiment called "Spring" allows users to simulate the behaviour of a spring-mass oscillation system. As students are familiar with the pendulum, and also with this spring-mass system from their physical laboratory classes, they should be able to deduce the system of differential equations. The last part of the course is related to the calculation of eigenvectors and eigenvalues, and it is very common that students do not understand the meaning of those ideas. Sometimes, for engineering students the "traditional" definition of eigenvector or proper vector of an endomorphism is more difficult that the concept of vibration modes of a spring, and the same for eigenvalues and frequencies.

With the use of appropriate mathematical language throughout the whole experiments the student acquires the competence of handling symbols and mathematical formalisms, which is closely related to communicating in, with and about mathematics, in which in the process of solving the tasks the student understands mathematical expressions and statements and expresses himself mathematically in different ways.

To solve any problem that arises to an engineering student, he must be able to use the necessary material and tools, which includes knowledge about the resources and tools that are available, as well as their potential and limitations. In addition, it includes the ability to use them carefully and efficiently.

The competencies related to the use of the technology are particularly relevant in engineering careers in general and in mathematics subjects in particular, since one of the causes of the difficulty in learning mathematics is the level of abstraction that these entail. The mobile phone is located precisely between the world of formal systems and the physical world and has the ability to make concrete the most abstract concepts (Turkle and Papert, 1992).

With the use of mathematics the student can experience solving mathematical problems through the computer. It is important to focus on two very important aspects of learning mathematics: the possibility of "experiencing" through the mobile devices and a change in the attitude of the student around the process of teaching and learning mathematics.

Approach of the Students: Problem of a building in vibration

After a computer-aided course we propose the students to solve a problem like this: Consider a two-story building in horizontal vibration by the action of the wind. The mechanical properties of the building are given by m_1 = 8000Kg, m_2 = 8000Kg, k_1 = 4000N/m and k_2 = 3500N/m. Calculate the natural modes and vibration frequencies (initial conditions are also given).

Reading this task, the student, who has knowledge of physics and is studying a engineering degree, can think that he must solve the problem from an algebraic equation and can give a mathematical answer (think mathematically). The relationship between the different forces involved in a system of two degrees of freedom must be translated into mathematical conditions or equations that include the mechanical data of the problem: the masses, m_1 and m_2, and the elastic constants, k_1 and k_2. To do this, he will apply a series of arguments (reasoning mathematically and representing mathematical entities) based on his physics' knowledge and the corresponding mathematical sense:

1. Thinking about the simplest system: a simple pendulum, its displacement is directly proportional to the force that produces it (Law of Hook): $F= k\ x$ (k = elastic constant, x = elongation).

2. The building will be at rest as long as there is no force acting on it that varies its initial state (Newton's first law). The use of drawings and graphs of problems helps to understand them (Domínguez Caicedo, 2014).

3. The forces produce accelerations that are proportional to the mass of a body. When there are several forces acting, they will be added vectorially (Newton's Second Law): $F = m\,x''$ (m = body mass, x''= acceleration).

A building can be modelled assuming that the walls do not have mass and that the mass is concentrated in the floors, so that there is a horizontal rigidity (Rao and Yap, 2011). The problem is equivalent to that of 2 springs and masses, as can be seen in Figure 3.

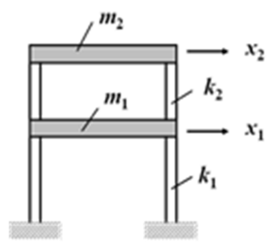

Figure 3. Two-storey building equivalent to 2 springs and masses.

The system of differential equations that models the system can be easily obtained from any textbook or directly searching on the internet (modeling mathematically), and the characteristic equation that will allow to get the eigenvalues (frequencies) and eigenvectors (vibration modes).

Moreover, the use of a computer algebra system, such as Mathematica could help to find the results quickly (Making use of aids and tools). With the use of appropriate mathematical language throughout the whole problem the student acquires the competence of handling mathematical symbols and formalisms, which is closely related to communicating in, with and about mathematics, in which in the process of solving the problem the student understands mathematical expressions and statements and expresses himself mathematically in different ways.

Once the problem is solved, a representation of the two-storey building vibrating could help to understand the meaning of the values of eigenvectors (see Figure 4).

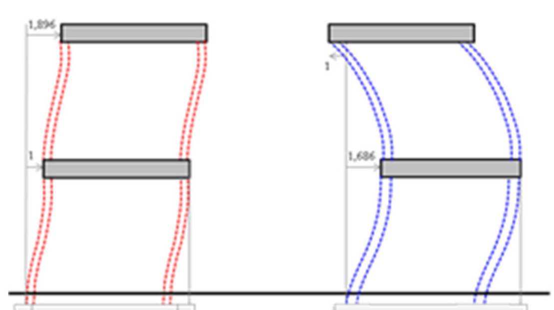

Figure 4. Final solution of the problem: 2 vibration modes.

Conclusions for Education

We started with the proposal of using the mobile phone as a spring together with a computer algebra system. Once students collect the data from their devices, they were able to calculate the frequency of a pendulum or a spring and this leads them to study the vibration of a system with one, two or more degrees of freedom. Mobile-aided learning and computer-based learning in general help students to be motivated. The use of devices awakens their curiosity and captures their attention.

While the traditional methodology starts from a formal and perfectly structured presentation of the contents and later focus on application problems, this methodology proposes to reverse the process: Initially, mathematization is induced through contextualized activities close to real life, sciences and engineering that are familiar to students, in order to later compare results and undertake the formal content fixing process.

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References

Alpers, B. et al. (2013) A Framework for Mathematics Curricula in Engineering Education. *SEFI*. Available at: http://sefi.htw-aalen.de/

Bullón, J.J., Hernández Encinas, A., Santos Sánchez, M.J., Gayoso Martínez, V. (2018) Analysis of student feedback when using gamification tools in Math subjects. In *Proc. IEEE Global Engineering Education Conference (EDUCON)*, 1824-1829.

Domínguez Caicedo, M. (2014) Períodos de vibración de las edificaciones. *Revista de Arquitectura e Ingeniería* 8(2), 1-13.

Rao, S.S. and Yap F.F. (2011) *Mechanical vibrations* vol. 4. Prentice Hall Upper Saddle River.

Turkle, S. y Papert, S. (1992) Epistemological pluralism and the revaluation of the concrete. *Journal of Mathematical Behavior* 11(1), 3-33.