Pergamon

Nonlinear Analysis, Theory, Methods & Applications, Vol. 30, No. 5, pp. 2873–2882, 1997
Proc. 2nd World Congress of Nonlinear Analysts

0362-546X/97 $17.00 + 0.00

PII: S0362-546X(97)00341-6

APPLICATION OF A NONLINEAR EVOLUTION MODEL TO FIRE PROPAGATION

R. MONTENEGRO,† A. PLAZA,† L. FERRAGUT,♦ and M.I. ASENSIO♦

†Department of Mathematics, University of Las Palmas de Gran Canaria, Las Palmas de Gran Canaria, Spain; and
♦Department of Pure and Applied Mathematics, University of Salamanca, Spain

Key words and phrases: Numerical Simulation, finite difference, adaptive finite element method.

1. INTRODUCTION

The numerical simulation of fire in forest has been an important objective in recent researches. The rate of spread and shape of a forest fire front is affected by many factors. The most important of these are as follows: fuel type and moisture content, wind velocity and variability, forest topography, fire spread mechanism, fuel continuity and the amount of spotting (cf.[1-2]). The development of Geographic Information Systems allows the incorporation of these data to the developed models. The first models took into account constant factors, continuous uniform fuel type, constant wind velocity, moisture and slope. Under these conditions, a fire ignited at a single point reaches a quasi-steady state and progresses toward the down wind direction and expands at a constant rate. These data cannot give precise predictions under variable conditions but are very useful in order to the intuition of the fire controller. Models capable of being incorporated into the computer simulations of fires under variable conditions have been developed, based on cellular automata (cf. [3-7]), and stochastic process [8]. These models can give useful indicators as to fire behavior under such conditions. Combustion phenomena has been extensively studied [9], unsteady flame propagation has been analyzed [10]. Models based on combustion theories are very difficult to develop because of the diversity of the fuel type and varied chemical composition within a given fuel type. Because of the complexity of the problem, models based rigorously on combustion theory have not been completely developed. In this preliminary work, a first attempt is done to design a computer code for numerical simulation of forest fire spread in landscapes. Basically a convection-diffusion model for temperature and a mass-consistent model for wind field simulation will be assumed. A two-steps chemical mechanism is simplified in order to obtain the heat source. This proposed 2-D model take into account the convection phenomena due to temperature gradients in vertical direction. A numerical solution of the former model is presented using a finite difference method together with the study of stability. This numerical method is contrasted with an adaptive finite element method using refinement/derefinement techniques (cf. [11-14]).

2. MATHEMATICAL MODEL

In this section we present the mathematical model for fire simulation in 2-D. We consider turbulent flow, then the average value of temperature T, at a point $\vec{x}$ in time t, is given by the *energy equation*,

$$\frac{\partial T}{\partial t} + \vec{v}\cdot\vec{\nabla}T - \vec{\nabla}\cdot\left(K\vec{\nabla}T\right) + h(T)(T - T_\infty) = f(t,T) \tag{2.1}$$

where $\vec{v}$ express the average value of wind velocity; $K = \frac{k}{\rho c}$ is the turbulent diffusivity, with ρ the air density and c its specific heat; $h(T) = \frac{h^*(T)}{\rho c}$ is the vertical convection heat transfer coefficient (cf. [15-16]), with $h^*(T) = \overline{h}(T - T_\infty)^{\frac{1}{3}}$ being T_∞ the ambient temperature; $f(t,T)$ is the heat source due to combustion. For equation (2.1) we consider the corresponding initial and boundary conditions.

In order to define the value of the heat source we assume first a *two-steps chemical model*. Suppose that we have a solid fuel A which produces, in an endothermic reaction, a gas fuel B. After this step, the gas fuel B produces heat in an exothermic reaction. An outline of this two-steps chemical process is the following:

$$\text{A (solid fuel)} + \text{heat} \rightarrow \text{B (gas fuel)} \rightarrow \text{products} + \text{heat source}$$

The unitary reactant quantities, C_A and C_B, for each step are given by the law of mass action and Arrhenius law, such that

$$\frac{dC_A}{dt} = -\lambda_A C_A^m \;;\; \lambda_A = Ae^{\frac{-E_A}{RT}} \;;\; A = \bar{A}\sqrt{T} \;;\; C_A(0) = C_{A_0} \tag{2.2}$$

$$\frac{dC_B}{dt} = \lambda_A C_A^m - \lambda_B C_B^n \;;\; \lambda_B = Be^{\frac{-E_B}{RT}} \;;\; B = \bar{B}\sqrt{T} \;;\; C_B(0) = C_{B_0} \tag{2.3}$$

where λ_A and λ_B are the reaction rates given by the Arrhenius law, R is the universal gas constant, E_A and E_B represent the activation energies, A and B are the frequency factors for the first and second reaction step respectively, that depend weakly on T. With this two-step chemical model, the heat source term in equation (2.1) is given by

$$f(t,T) = Q_B \lambda_B C_B^n - (Q_A + Q_B)\lambda_A C_A^m \tag{2.4}$$

where $Q_I = \frac{q_I}{\rho c}$, and q_A is the unitary heat loss in the endothermic reaction and q_B is the unitary heat production in the exothermic reaction.

This model can be simplified in the following way. Assuming that the activation energy E_B is lower than E_A, then $\lambda_A << \lambda_B$, and considering that $m=n=1$ we have a simplified *one-step chemical model*:

$$\text{A} \rightarrow \text{products} + \text{heat source}$$

The unitary reactant quantities C_A for this step is given by:

$$\frac{dC_A}{dt} = -\lambda_A C_A \;;\; \lambda_A = Ae^{\frac{-E_A}{RT}} \;;\; A = \bar{A}\sqrt{T} \;;\; C_A(0) = C_{A_0} \tag{2.5}$$

and the heat source is $f(t,T) = Q\lambda_A C_A$, where $Q = \frac{q}{\rho c}$, is the global heat production in the reaction. This heat production assume that reaction take place in all the volume of the solid fuel. This is not always true. In other cases, for example if the combustion take place only on the surface, decay with time can be linear or parabolic, instead of the former exponential law.

In this preliminary work, the one-step chemical model is assumed, but in a future work the two-steps model will be considered and endothermic effects corresponding to the first step will be taken into account.

In order to simulate the *wind velocity field* a mass consistent model will be adopted [13]. This model has considerable advantages with respect to others, like primitive equations models, due to simplicity of the method for solving the differential equations governing the flow, and hence short computational time, and small number of input data. A brief description of the objective of the model is the following:

Let $\Omega \subset \mathrm{R}^d$ be a bounded domain with boundary $\partial\Omega = \Gamma_1 \cup \Gamma_2$. Suppose that we construct from the interpolation/extrapolation of experimental data a wind field $\vec{v}_0$ defined in all the domain. Then we look for a vectorial field $\vec{v}$ that adjusts, in a least square sense, to the field $\vec{v}_0$, verifying the incompressibility condition and the impermeable boundary Γ_1, that is

$$\vec{\nabla}\cdot\vec{v}=0 \quad \text{in}\,\Omega \tag{2.6}$$

$$\vec{v}\cdot\vec{n}=0 \quad \text{in}\,\Gamma_1 \tag{2.7}$$

where $\vec{n}$ is the vector, unitary and perpendicular to Γ_1. The vectorial field $\vec{v}$ is the solution of the problem:

"Find $\vec{v}\in K$ that verifies,

$$\begin{cases} J(\vec{v})=\min_{\vec{\mu}\in K} J(\vec{\mu}) \\ K=\left\{\vec{\mu};\ \vec{\nabla}\cdot\vec{\mu}=0,\ \vec{\mu}\cdot\vec{n}\big|_{\Gamma_1}=0\right\} \end{cases} \tag{2.8}$$

where the cost function is

$$J(\vec{\mu})=\tfrac{1}{2}\int_\Omega(\vec{\mu}-\vec{v}_0)^t P(\vec{\mu}-\vec{v}_0)d\Omega \tag{2.9}$$

This problem can be formulated as a saddle-point problem for a Lagrangian. Details about the adaptive mixed finite element method, developed for the solution of this problem, can be seen in [13]. With this method we get an exactly divergence-free wind field.

In practice, the following nondimensional version of the presented mathematical model for fire simulation will be solved

$$\frac{\partial u}{\partial\tau}+\vec{\beta}\cdot\vec{\nabla}u-\vec{\nabla}\cdot\left(\frac{1}{P_e}\vec{\nabla}u\right)+\tilde{h}(u)u=\tilde{f}(\tau,u) \tag{2.10}$$

$$\frac{dN}{d\tau}=-\frac{a}{|\vec{v}_0|}\lambda_A N\ ;\ \lambda_A=Ae^{\frac{-E_A}{R\left[T_\infty+u(\tilde{T}-T_\infty)\right]}} \tag{2.11}$$

where nondimensional space $\vec{\zeta}$ and time τ have been introduced, such that: $\vec{x}=a\vec{\zeta}$, $\tau=|\vec{v}_0|t/a$, a is a significant distance of the domain and $|\vec{v}_0|$ is the maximum module of the wind velocity along the domain; $P_e=a|\vec{v}_0|/K$ is the Peclet number; $\vec{\beta}=\vec{v}/|\vec{v}_0|$ is the nondimensional wind velocity field; similarly $u=(T-T_\infty)/(\tilde{T}-T_\infty)$, with $\tilde{T}$ as a temperature up to which generalized combustion taken place; $N=C_A/C_{A_0}^M$ is the normalized fuel, where $C_{A_0}^M$ is chosen as the maximum initial fuel along the domain; the normalized vertical convection coefficient and the normalized heat source are given by

$$\tilde{h}(u)=\frac{a}{|\vec{v}_0|}\frac{\bar{h}}{\rho c}\left[(\tilde{T}-T_\infty)u\right]^{\frac{1}{3}} \tag{2.12}$$

$$\tilde{f}(\tau,u)=\frac{a}{|\vec{v}_0|(\tilde{T}-T_\infty)}QAC_{A_0}^M Ne^{\frac{-E_A}{R\left[T_\infty+u(\tilde{T}-T_\infty)\right]}} \tag{2.13}$$

In nondimensional model we will have to introduce the initial conditions for u and N, that is the initial distribution of the normalized temperature and fuel, and the boundary condition for u. We can take as boundary condition the ambient temperature u=0, if the fire is far from the boundary.

3. FINITE DIFFERENCE ANALYSIS

In this section we study the problem presented from (2.10) to (2.13) and particularized in 2-D with a wind field velocity and diffusivity constant along the domain. Therefore, equation (2.10) can be written

$$\frac{\partial u}{\partial \tau}+\beta_\xi \frac{\partial u}{\partial \xi}+\beta_\eta \frac{\partial u}{\partial \eta}-\frac{1}{P_e}\left(\frac{\partial^2 u}{\partial \xi^2}+\frac{\partial^2 u}{\partial \eta^2}\right)+\tilde{h}(u)u=\tilde{f}(\tau,u) \tag{3.1}$$

Suppose that the numerical values, for all grid points at a general time step, are known; $u_{i,j}^n \approx u(\xi_i,\eta_j,\tau_n)$ and $N_{i,j}^n \approx N(\xi_i,\eta_j,\tau_n)$. Then, In order to obtain the numerical solution of the normalized temperature in the following time step, $\tau_{n+1}=\tau_n+\Delta\tau$, we use an implicit and upwind finite difference scheme for energy equation (3.1)

$$\mathrm{dyl}\,u_{i,j-1}^{n+1}+\mathrm{dxl}\,u_{i-1,j}^{n+1}+\mathrm{dp}\,u_{i,j}^{n+1}+\mathrm{dyu}\,u_{i,j+1}^{n+1}+\mathrm{dxu}\,u_{i+1,j}^{n+1}=\Delta\tau\,\tilde{f}\left(\tau_n,u_{i,j}^n\right)+u_{i,j}^n \tag{3.2}$$

where

$$\tilde{h}(u_{i,j}^n)=\frac{a}{|\vec{v}_0|}\frac{\bar{h}}{\rho c}\left[(\tilde{T}-T_\infty)u_{i,j}^n\right]^{\frac{1}{3}}$$

$$\tilde{f}(\tau_n,u_{i,j}^n)=\frac{a}{|\vec{v}_0|(\tilde{T}-T_\infty)}QA_{i,j}^n C_{A_0}^M N_{i,j}^n e^{\frac{-E_A}{R\left[T_\infty+(\tilde{T}-T_\infty)u_{i,j}^n\right]}}$$

$$\mathrm{dp}=1-\mathrm{dyl}-\mathrm{dxl}-\mathrm{dyu}-\mathrm{dxu}+\Delta\tau\,\tilde{h}(u_{i,j}^n)$$

and according to the direction of the normalized wind velocity field, $\vec{\beta}=\beta_\xi\,\vec{i}+\beta_\eta\,\vec{j}$, we obtain that

If $\beta_\xi \geq 0$ and $\beta_\eta \geq 0$

$$\mathrm{dyl}=-\frac{\Delta\tau}{P_e\Delta\eta^2}-\frac{\beta_\eta\Delta\tau}{\Delta\eta},\quad \mathrm{dxl}=-\frac{\Delta\tau}{P_e\Delta\xi^2}-\frac{\beta_\xi\Delta\tau}{\Delta\xi},\quad \mathrm{dyu}=-\frac{\Delta\tau}{P_e\Delta\eta^2},\quad \mathrm{dxu}=-\frac{\Delta\tau}{P_e\Delta\xi^2}$$

If $\beta_\xi \leq 0$ and $\beta_\eta \geq 0$

$$\mathrm{dyl}=-\frac{\Delta\tau}{P_e\Delta\eta^2}-\frac{\beta_\eta\Delta\tau}{\Delta\eta},\quad \mathrm{dxl}=-\frac{\Delta\tau}{P_e\Delta\xi^2},\quad \mathrm{dyu}=-\frac{\Delta\tau}{P_e\Delta\eta^2},\quad \mathrm{dxu}=-\frac{\Delta\tau}{P_e\Delta\xi^2}+\frac{\beta_\xi\Delta\tau}{\Delta\xi}$$

If $\beta_\xi \leq 0$ and $\beta_\eta \leq 0$

$$\mathrm{dyl}=-\frac{\Delta\tau}{P_e\Delta\eta^2},\quad \mathrm{dxl}=-\frac{\Delta\tau}{P_e\Delta\xi^2},\quad \mathrm{dyu}=-\frac{\Delta\tau}{P_e\Delta\eta^2}+\frac{\beta_\eta\Delta\tau}{\Delta\eta},\quad \mathrm{dxu}=-\frac{\Delta\tau}{P_e\Delta\xi^2}+\frac{\beta_\xi\Delta\tau}{\Delta\xi}$$

If $\beta_\xi \geq 0$ and $\beta_\eta \leq 0$

$$\mathrm{dyl}=-\frac{\Delta\tau}{P_e\Delta\eta^2},\quad \mathrm{dxl}=-\frac{\Delta\tau}{P_e\Delta\xi^2}-\frac{\beta_\xi\Delta\tau}{\Delta\xi},\quad \mathrm{dyu}=-\frac{\Delta\tau}{P_e\Delta\eta^2}+\frac{\beta_\eta\Delta\tau}{\Delta\eta},\quad \mathrm{dxu}=-\frac{\Delta\tau}{P_e\Delta\xi^2}$$

The matrix of the linear system in each time step is block tridiagonal and it verifies: diagonal terms are strictly greater than zero, other terms in rows are less or equal than zero and the matrix is strictly diagonally

dominant. Therefore, this matrix is an M-matrix, then the blockwise Gauss-Seidel method converge and it is faster than the point-wise method [17].

In order to obtain the numerical solution of the normalized fuel in time step, $\tau_{n+1} = \tau_n + \Delta\tau$, we use an implicit Euler method for fuel equation (2.11)

$$N_{i,j}^{n+1} = \frac{N_{i,j}^{n}}{1 + \Delta\tau \frac{a}{|\vec{v}_0|} A_{i,j}^{n+1} e^{\frac{-E_A}{R\left[T_\infty + (\tilde{T} - T_\infty) u_{i,j}^{n+1}\right]}}} \tag{3.3}$$

For the analysis of stability of the finite difference model given by (3.2) and (3.3) we take into account that equations (2.11) and (3.1) have been uncoupled. Stability of the implicit Euler scheme (3.3) is well known. On the other hand, we can obtain a stability condition for energy equation scheme (3.2) due to a global Lipschitz condition that the source term (2.13) verify. First, let us consider the following theorem in a more general case.

THEOREM 3.1. If we have the following general problem:

$$\begin{aligned} &\frac{\partial u}{\partial t} - \sum_{i,j=1}^{d} \frac{\partial}{\partial x_i}\left(a_{i,j} \frac{\partial u}{\partial x_j} \right) + b_0 u = \sum_{i=1}^{d} b_i \frac{\partial u}{\partial x_i} + f(u) \qquad in\ \Omega,\ \forall t \geq 0 \\ &u = 0 \qquad in\ \partial\Omega,\ \forall t \geq 0 \\ &u(\vec{x}, 0) = g(\vec{x}) \qquad in\ \Omega \end{aligned} \tag{3.4}$$

where $a_{i,j}$, b_i are smooth bounded functions of $\vec{x}$, t, such that

$$\forall \vec{\varsigma} \in \Re^d,\ \exists \gamma > 0\ /\ \sum_{i,j=1}^{d} a_{i,j} \varsigma_i \varsigma_j \geq \gamma \sum_{i=1}^{d} (\varsigma_i)^2$$

$$\exists B > 0\ /\ |b_i| \leq B \qquad (i = 1, \ldots, d)$$

and function *f(u)* verify a global Lipschitz condition,

$$\exists L > 0\ /\ |f(u_1) - f(u_2)| \leq L|u_1 - u_2| \qquad \forall u_1, u_2 \in Range(u)$$

Then, the finite difference scheme:

$$\partial_t u^n - \sum_{i,j=1}^{d} \partial_{x_i}\left(a_{i,j}^n \bar{\partial}_{x_j} u^{n+1} \right) + b_0^n u^{n+1} = \sum_{i=1}^{d} b_i^n \partial_{x_i} u^{n+1} + f(u^n)$$

using the following notation

$$\partial_{x_i} \vartheta = \frac{\vartheta_{i+1} - \vartheta_i}{\Delta x_i} \ ;\ \bar{\partial}_{x_i} \vartheta = \frac{\vartheta_i - \vartheta_{i-1}}{\Delta x_i} \ ;\ \partial_t \vartheta^n = \frac{\vartheta^{n+1} - \vartheta^n}{\Delta t}$$

is stable if $\Delta t \leq \dfrac{1}{\frac{d\,B^2}{\gamma} + \sqrt{2}L + 2}$

and we get the following error estimate

$$\left\|e^n\right\|^2 \le e^{nC\Delta t}\left\|e^0\right\|^2 + 2\Delta t\sum_{k=1}^{n}\left\|\sigma^k\right\|^2$$

where $\sigma^k = O(\Delta t) + \sum_{i=1}^{d} O(\Delta x_i)$ is the consistency error in time $t_k = k\Delta t$ and $C = \frac{d\,B^2}{\gamma} + 2\sqrt{2}L + 1$.

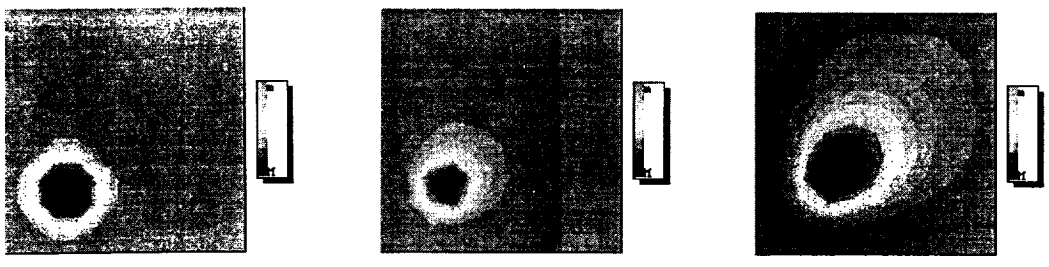
Based on this theorem, we can prove the conditional stability of scheme (3.2) because the heat source term verify a global Lipschitz condition with

$$L = \frac{a}{|\vec{v}_0|}Q\bar{A}\,C_{A_0}^{M}\frac{\sqrt{R}}{\sqrt{E_A}}\frac{1}{2\left(\sqrt{2}-1\right)\sqrt{\sqrt{2}-1}}e^{-\frac{1}{2\left(\sqrt{2}-1\right)}}$$

Let us consider the a problem as a numerical example of the finite difference method proposed in this section. Suppose a square domain Ω = [0,a]×[0,a], with a=6, $\beta_\xi = \beta_\eta = \sqrt{2}/2$, $\tilde{T} = 1000$, $T_\infty = 300$, $1/P_e = 0.005$, $Q = 1583.3$, $E_A = 20000$, $R = 1.97817$, $\rho = 1$, $c = 1000$, $\bar{h} = 1.43$, $\bar{A} = 7500$. We take the following initial condition for the normalized temperature

$$u(\vec{\zeta},0) = \begin{cases} d_1 & \text{if } \left|\vec{\zeta}-\vec{\zeta}_0\right| < R_1 \\ d_2 & \text{if } \left|\vec{\zeta}-\vec{\zeta}_0\right| > R_2 \\ d_2 + (d_1 - d_2)e^{\frac{R_1^2 - \left|\vec{\zeta}-\vec{\zeta}_0\right|^2}{R_2^2 - \left|\vec{\zeta}-\vec{\zeta}_0\right|^2}} & \text{if other} \end{cases}$$

where $\vec{\zeta} = (0.25, 0.25)$, $R_1 = 0.1$, $R_2 = 0.2$, $d_1 = 1.5$, $d_2 = 0$. A constant initial fuel distribution $N(\vec{\zeta},0) = 1$ is assumed in all the domain. Finally, we suppose a Dirichlet boundary condition $u = 0$ in $\partial\Omega$, $\forall\tau \ge 0$. The temperature distribution for three time steps can be seen in Fig. 1. The values of the numerical solution can observed in the grey scale from the maximum value M=1350 °K to the minimum one m=300 °K.



(a) (b) (c)

Fig. 1. Temperature distribution for τ=0 (a), τ=0.1 (b) and τ=0.2 (c).

4. ADAPTIVE FINITE ELEMENT METHOD

To contrast results obtained with the former finite difference model, we propose to use an adaptive finite element method described in [12]. This method combine refinement and derefinement techniques to generate in each time step a sequence of nested meshes allowing an easy application of multigrid acceleration techniques. With this method we can adapt the mesh to the numerical solution where more precision is necessary. We propose to use the finite difference scheme (3.3) for solving the fuel equation in each node of the mesh and to use finite element method for the energy equation (2.10). The time discretization of this equation can be an extension of the one presented in [11] and [18] for a linear convection-diffusion problem. This process can be summarized as follows for the nonlinear problem. Equation (2.10) can be written as

$$\frac{du}{d\tau} - \vec{\nabla}\cdot\left(\frac{1}{P_e}\vec{\nabla}u\right) + \tilde{h}(u)u = \tilde{f}(\tau,u) \tag{4.1}$$

Let be a fluid element at point $\underline{P}(\vec{\zeta},\tau_n)$. After a time step, this fluid element will be in the position $P(\vec{\zeta},\tau_{n+1})$. Using the following approximation

$$\frac{du}{d\tau} \approx \frac{u(\vec{\zeta},\tau_{n+1}) - u(\underline{\vec{\zeta}},\tau_n)}{\Delta\tau} = \frac{u^{n+1}(\vec{\zeta}) - u^n(\underline{\vec{\zeta}})}{\Delta\tau}$$

to do an Euler semi-implicit approximation in (4.1), we obtain

$$u^{n+1}(\vec{\zeta}) - \Delta\tau\,\vec{\nabla}\cdot\left[\frac{1}{P_e}\vec{\nabla}u^{n+1}(\vec{\zeta})\right] + \Delta\tau\,\tilde{h}(u^n(\vec{\zeta}))u^{n+1}(\vec{\zeta}) = \Delta\tau\tilde{f}(u^n(\vec{\zeta})) + u^n(\underline{\vec{\zeta}}) \tag{4.2}$$

In equation (4.2) we should have the problem to evaluate the last term in the discrete domain. In order to solve this problem, we try to write this equation depending only of what happen at the point P at every time. Then, we can approximate

$$\underline{\zeta}_i = \zeta_i(\tau_{n+1} - \Delta\tau) = \zeta_i - \beta_i(\vec{\zeta})\Delta\tau + \frac{\Delta\tau^2}{2}\vec{\beta}(\vec{\zeta})\cdot\vec{\nabla}\beta_i(\vec{\zeta}) + 0(\Delta\tau^3)$$

and write

$$u^n(\underline{\vec{\zeta}}) = u^n(\vec{\zeta}) - \Delta\tau\sum_{i=1}^{2}\beta_i(\vec{\zeta})\frac{\partial u^n(\vec{\zeta})}{\partial\zeta_i} + \frac{\Delta\tau^2}{2}\sum_{i=1}^{2}\left[\vec{\beta}(\vec{\zeta})\cdot\vec{\nabla}\beta_i(\vec{\zeta})\right] + \frac{\Delta\tau^2}{2}\sum_{i=1}^{2}\sum_{j=1}^{2}\beta_i(\vec{\zeta})\beta_j(\vec{\zeta})\frac{\partial^2 u^n(\vec{\zeta})}{\partial\zeta_i\partial\zeta_j} + 0(\Delta\tau^3)$$

Finally, if we introduce this last expression in (4.2) we get the following semi-implicit formulation that approximate the energy equation (2.10), where all terms are evaluated at the same spatial coordinate of P

$$u^{n+1} - \Delta\tau\,\vec{\nabla}\cdot\left[\frac{1}{P_e}\vec{\nabla}u^{n+1}\right] + \Delta\tau\,\tilde{h}(u^n)u^{n+1} = u^n + \Delta\tau f(u^n) - \Delta\tau\,\vec{\beta}\cdot\vec{\nabla}u^n + \frac{\Delta\tau^2}{2}\sum_i(\vec{\beta}\cdot\vec{\nabla}\beta_i)\frac{\partial u^n}{\partial\zeta_i} + \frac{\Delta\tau^2}{2}\sum_{i,j}\beta_i\beta_j\frac{\partial^2 u^n}{\partial\zeta_i\partial\zeta_j}$$

Now, considering boundary conditions, it is easy to obtain the variational formulation and then apply the finite element method. A stability and consistency study of this formulation in the linear case can be seen in [18]. The adaptive strategy is presented also in this reference.

In order to show the efficiency of the adaptive process we consider a similar numerical example, in a rectangular domain, to the one presented in the previous section. Here we take an uniform wind velocity field parallel to the longest side of the rectangular domain. The initial temperature distribution is shown in Fig. 2 (a) and a detail of the adaptive mesh corresponding to this initial solution can be observed in Fig. 2 (b). The adaptive strategy enable us to get a good approximation with a minimum number of nodes. The meshes for

four following time steps can be seen in Fig. 2 (c)-(f). It is worth to be noted that the number of nodes in the meshes are bounded during the unsteady process.

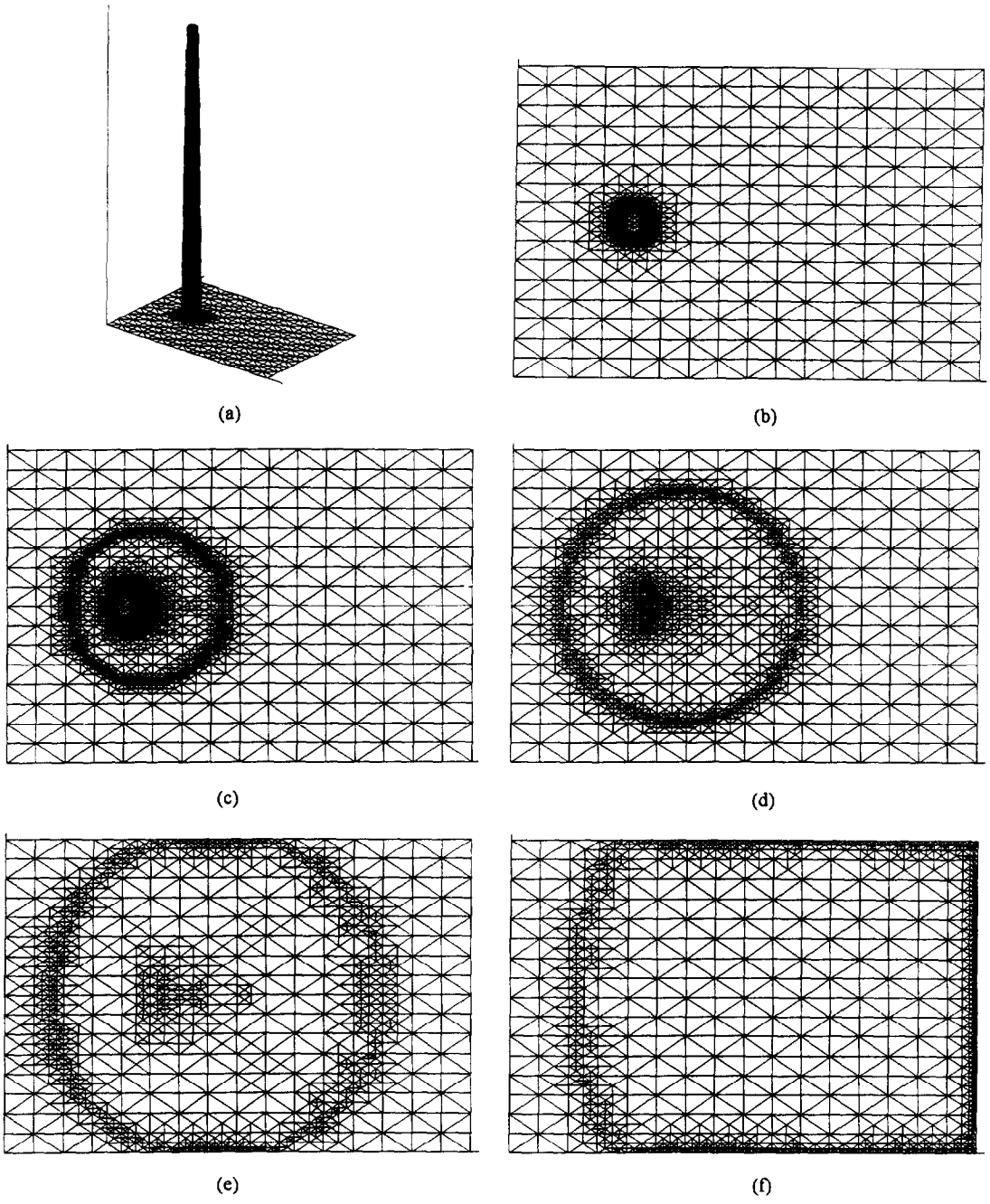


Fig. 2. Initial temperature distribution (a) and the corresponding mesh (b). Meshes for four following time steps (c)-(f).

5. CONCLUSIONS AND FUTURE RESEARCHES

A model to simulate forest fire is proposed in order to develop a computer code. Though a simplified mathematical model is assumed, the principal fire spread mechanisms are taken into account: turbulent diffusion and convection. The heat loss by convection phenomena in the vertical direction is also taken into account by means of an estimated convection heat transfer coefficient. Topographic effects of slope affects the spread of fire, modifying the value of this coefficient. Others effects, like fuel moisture and the type of the solid fuel, affect the value of the parameters of the model which must be adjusted case by case. A procedure to investigate ignitions through time and space is also proposed. The results of that computation will be used to calibrate the model.

For simple case, that is, nearly homogeneous fuel distribution, steady state wind conditions, etc. the finite difference method presented in this work, including a stability analysis, seems to be very efficient, allowing numerical fire simulation at low computational cost. On the contrary, when high heterogeneities are present and we have a solution with singularities or boundary layers the automatic adaptive multigrid technique mentioned in this work has proved to be useful in order to obtain high quality numerical solutions, but at a high computational cost.

As future researches we think to implement the two-steps chemical model presented in this work. We also will treat the 3-D problem using the adaptive finite element method. A previous work about the 3-D mesh generation can be seen in [19]. We will extend the stability analysis of the linear convection-diffusion formulation to the nonlinear energy equation attending to the global Lipschitz condition of the source term.

6. ACKNOWLEDGMENT

This research has been sponsored in part by Junta de Castilla y León (Spain), Project SA50/95.

REFERENCES

1. GWYNFOR D.R., Numerical simulation of forest fires, *Int. J. Num. Meth. Eng.* **25**, 625-633 (1988).
2. ROTHERMEL R.C., A mathematical model for predicting fire spread in wildland fuels, *Research Paper INT-115*, USDA Forest Service (1972).
3. BAK P. &CHEN K., A forest-fire model and some thoughts on turbulence, *Physics Letters A* **147**, 297-300 (1993).
4. DROSSEL B. & SCHWABL F., Self-organized critical forest-fire model, *Physical Review Letters* **69**, 1629-1632 (1992).
5. PACZUSKI M. & BAK P., Theory of the one-dimensional forest-fire model, *Physical Review E* **48**, R3214-R3216 (1993).
6. GRASBERGER P., On a self-organized forest-fire model, *J. Physics A: Math. Gen.*, **26** 2081-2089 (1993).
7. PATZLAFF H. & TIMPER S., Analytical approach to the forest-fire model, *Physics Letters A* **189**, 187-192 (1993).
8. ZHANG Y., A shape theorem for epidemics and forest fires with finite range iteration, *The Annals of Probability* **21**, 1750-1781 (1993).
9. WILLIAMS F.A., Combustion Theory, *Addison-Wesley Publishing Company*, 2nd edn., (1985).
10 LARROUTUROU B., The equation of the one-dimensional unsteady flame propagation: Existence and uniqueness, *SIAM J. Math. Anal.* **19**, 32-59 (1988).
11. MONTENEGRO R., MONTERO G., FERRAGUT L. & WINTER G., Application of adaptive finite element methods to convection-diffusion problems in 2-D, (in Spanish), *Rev. Int. Met. Num. Ing.* **5**, 535-560 (1989).
12. FERRAGUT L., MONTENEGRO R. & PLAZA A., Efficient refinement/derefinement algorithm of nested meshes to solve evolution problems, *Comm. Num. Meth. Eng.* **10**, 403-412 (1994).
13. WINTER G., MONTERO G, FERRAGUT L. & MONTENEGRO R., Adaptive strategies using standard and mixed finite elements for wind field adjustment, *Solar Energy* **54**, 49-56 (1995).
14. RATTO C.F. et al., Mass-consistent models for wind fields over complex terrains: The state of the art, *Environmental Software* **9**, 247-268 (1994).
15. HOLMAN J.P., Heat Transfer, McGraw-Hill, 4th edn. (1976).
16. BARROW, Química Física, Reverté, 2nd edn. (1962).
17. HACKBUSH W., Iterative solution of large sparse system of equations, Springer-Verlag (1994).
18. PLAZA A., MONTENEGRO R. & FERRAGUT L., An adaptive refinement/derefinement algorithm of structured grids for solving time-dependent problems, in *Numerical Methods in Engineering'92* (Edited by Ch. HIRSCH, O.C. ZIENKIEWICZ and E. OÑATE), pp. 225-232. Elsevier Science Publishers B.V., Amsterdam (1992).

19. ESCOBAR J.M. & MONTENEGRO R., Several aspects of the three-dimensional triangulation of Delaunay, in *Advances in Post and Preprocessing for F.E.M.* (Edited by M. PAPADRAKAKIS and B.H.V. TOPPING), pp. 193-204. Edinburgh, Scotland (1994).