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EXTENDED SUBCRITICAL LIMITS FOR SLIGHTLY ENRICHED UO2
(≤ 5 WT % U-235)

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0. INTRODUCTION

1. GENERAL DESCRIPTION OF THE METHOD

2. THE PROCESS TO STABLISH MAXIMUM PERMISSIBLE VALUES

3. CRITERIA FOR INCREASING MPV

4. SUMMARY
0. INTRODUCTION

The Nuclear Criticality Safety (NCS) of most processing operation may be established using the suitable maximum permissible values (MPV) for simple systems.

The regulations of NCS (ref.1) do not define strict rules to establish these MPVs, but they indicate that the values of the parameters ("Subcritical limits"), must be reduced to an amount that quite enough assures the subcriticality.

At ENUSA's Juizbado Fuel Fabrication Plant (SPAIN) we have been using a conservative criterion, defining the MPV as the parameter values which can lead to a K-eff of 0.95 at top. The MPVs are calculated for very restrictive conditions and, on many occasions, when analysing a specific plant is convenient to use criterion for increasing this MPV.

In this paper we describe the method used to calculate MPVs and the way to increase this. The results are represented in tables and figures of very easy use.

In all cases we suppose mixture UO$_2$-H$_2$O enriched 5 wt% $^{235}$U.
1. GENERAL DESCRIPTION OF THE METHOD

The method used to determine the MPVs is the one that follows:

1°. Find the minimum critical values of the parameters using the data of the ref. 2 and 3.

2°. Obtain, using again the ref. 2 and 3, $M^2$, $\delta$ and $K^m$.

3°. Using the ecs 1, 2, 3 and 4 find the parameters that corresponds to $K_{eff} = 0.95$. In all cases we suppose optimum moderation and full reflection.

The obtained results can be seen in table 1. The detailed calculations are described in the 2-nd section.

<table>
<thead>
<tr>
<th>MPV OF MIXTURE (UO₂ + H₂O)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MPV</strong></td>
</tr>
<tr>
<td>- MASS</td>
</tr>
<tr>
<td>- VOLUME</td>
</tr>
<tr>
<td>- RATIO OF</td>
</tr>
<tr>
<td>INFINITE CYLINDER</td>
</tr>
<tr>
<td>THICKNESS OF</td>
</tr>
<tr>
<td>INFINITE SLAB</td>
</tr>
</tbody>
</table>

a) **Sphere**

\[
(1) Bg^2 (sphere) = \frac{n^2}{(R_s + \delta_s)^2}
\]

b) **Cylinder of finite length**

\[
(2) Bg^2 (cylinder) = \frac{(2.405)^2 + \frac{n^2}{(R_c + \delta_c)^2}}{(H + 2\delta_c)^2}
\]
c) **Slab with finite dimensions**

\[
B_g^2 \text{ (slab)} = \frac{n^2}{(T+2\delta_t)^2} + \frac{n^2}{(L+2\delta_t)^2} + \frac{n^2}{(W+2\delta_t)^2}
\]

\[K_s = \frac{K_{ef}}{M^2}\]

\[B_g^2 = \frac{K_s - K_{ef}}{K_{ef} M^2}\]

Where:

- \(R_s\) = Radius of a sphere
- \(T\) = Finite slab thickness
- \(L\) = Finite slab length
- \(W\) = Finite slab width
- \(H\) = Finite cylinder height
- \(\delta_s\) = Extrapolation distance of a full reflected sphere
- \(\delta_t\) = The same for a slab
- \(\delta_c\) = The same for a cylinder
- \(M^2\) = Migration area
- \(K_{ef}\) = We always choose 0.95 as a criterium to establish MPV

Besides we have increased these values limiting some additional parameters such as: relationship \(H/U\), slab length, slab width, cylinder height. The results are shown in figs. 1 to 6. The detailed calculations are described in 3-rd section.

We have verified the methods above mentioned making some additional calculations with the Scale System. They prove the validity of the used method.
2. THE PROCESS TO ESTABLISH MAXIMUM PERMISIBLE VALUES (MPV)

2.1. MPV FOR HOMOGENEOUS SYSTEMS

a) MPV for Mass

Using tables of ref. 2, we obtain a density, \( d \), 1.000 g. U/liter for minimum critical mass and for this density and full reflection, we find:

- \( R_m = 1.39 \)
- \( M^2 = 28.5 \text{ cm}^2 \) and
- \( \delta_s = 6 \text{ cm} \)

Of eqs (4) and (1) we obtain:

\[
\pi \frac{R_s}{\left(1.39-0.95\right)/ \left(0.95 \cdot 28.5\right)} = 6 = 18.65 \text{ cm}
\]

So,

\[
\text{MPV for mass} = \frac{4\pi}{3} 18.64^3 \text{ cc.} \quad 1 \text{ g U/cc} = 27.14 \text{ g U} = 31 \text{ Kg UO}_2
\]

b) MPV for Volume

Using ref. 2 and eqs 1 and 4 we find:

- Minimum critical radius = 18.41 cm
- \( d = 2100 \text{ g U/l} \)
- \( R_m = 1.44 \)
- \( M^2 = 28 \text{ cm}^2 \)
- \( \delta = 6.45 \text{ cm} \)

\[
\pi \frac{R}{\left(1.44-0.95\right)/ \left(0.95 \times 28\right)} = 6.45 = 16.7 \text{ cm}
\]

\[
V = \frac{4}{3}\pi R^3 = 19,756 \text{ cc} = 19.75 \text{ liters}
\]
c) **MPV for infinite cylinder**

Using tables of ref. 2, we obtain a density, \( d \), 2.100 g U/l for minimum radius infinite cylinder and full reflection, we ind:

- \( K_\infty = 1.44 \)
- \( M^2 = 28 \text{ cm}^{-2} \)
- \( \delta = 6.45 \text{ cm} \)

of ecs (2) and (4) results

\[
R_c = \frac{2.405}{(1.44-0.95)/0.95\times28} \approx 6.45 = 11.2 \text{ cm}
\]

**d) MPV for a infinite slab**

The density for a minimum critical slab is 2.100 g. U/l (ref. 2). For this density,

- \( K_\infty = 1.44 \)
- \( M^2 = 28 \text{ cm}^{-2} \)
- \( \delta = 6.45 \text{ cm} \)

of ecs (3) and (4) we obtain,

\[
L = \frac{\pi^2}{(1.44-0.95)/(0.95\times28)} \approx 2 \times 6.45 = 10.25 \text{ cm}
\]
2.2. MPV FOR HETEROGENEOUS SYSTEMS

Following the same method but using the tables of ref.3 for heterogeneous system we obtain these MPVs.

a) MPV for mass

- Minimum critical mass: 1560 g. $^{235}\text{U}$
- Rods diameter: 0.254 cm
- $M^2 = 29.77 \text{ CM}^{-2}$
- $d = 6.29 \text{ cm}$
- $d (^{235}\text{U}) = 47.32 \text{ g. } ^{235}\text{U/1}$

- The sphere volume for minimum critical mass:

\[
\frac{\text{minimum critical mass } ^{235}\text{U}}{\text{density } ^{235}\text{U}} = \frac{1560}{47.32} = 32.97 \text{ liters}
\]

- The sphere radius for minimum critical mass:

\[
R_c = \left(\frac{3 \cdot V_c}{4}\right)^{1/3} = 19.89 \text{ cm}
\]

With these parameters and using eqs (1) and (4) for $\text{Kef} = 1$

\[
K = 1 + \frac{29.77 \text{ m}^2}{(19.89 + 6.29)^2} = 1.43
\]

if $\text{Kef}: 0.95$

\[
R = \frac{\pi}{\left[(1.43-0.95)/ 1.43 \times 29.77\right]^{1/3}} - 6.29 = 17.86
\]

\[
\text{MPV for Mass} = \frac{4\pi}{3} (17.86)^3 47.32 = 1.128 \text{ g. } ^{235}\text{U} = 22 \text{ kg. U} = 25 \text{ kg UO}_2.
\]
b) **MPV for volume**

- Minimum critical volume = 22.1 l.
- Rods Diameter = 0.762 cm
- \( M^2 = 30.15 \text{ cm}^{-2} \)
- \( \delta = 6.57 \text{ cm} \)
- \( d (^{238}\text{U}) = 118.56 \text{ g } ^{235}\text{U}/\text{liters} \)

Substituting these values in the ecs (1) and (4), for \( K_{ef} = 1 \)

\[
K = 1 + \frac{M^2 \pi^2}{(R_c + 6.29)^2} = 1 + \frac{M^2 \pi^2}{3 \times 22.1 \left\{ \left( \frac{\pi}{4} \right)^{1/3} + 6.29 \right\}^2} = 1.518
\]

if \( K_{ef}: 0.95 \)

\[
R = \frac{\pi}{\left\{ (1.518 - 0.95)/0.95 \times 30.15 \right\}^{1/2}} - 6.57 = 15.75
\]

\[
\text{MPV for Volume} = \frac{4\pi}{3} 15.75^3 \approx 16 \text{ l.}
\]

c) **MPV for infinite cylinder**

- Minimum critical radius = 17.7 cm
- Rods Diameter = 0.762 cm
- \( M^2 = 30.15 \text{ cm}^{-2} \)
- \( d (^{238}\text{U}) = 118.56 \text{ g } ^{235}\text{U}/\text{liter} \)
- \( \delta = 6.63 \text{ cm} \)

Substituting in ecs (1) and (3) for \( K_{ef} = 1 \)

\[
K = 1 + \frac{M^2 \pi^2}{(R_c + \delta)^2} = 1.519 \quad K = 1 + \frac{30.15 \times 2.405^2}{(11.7 + 6.63)}
\]
If $\text{Keff.} = 0.95$

$R_c = [2.40/[(1.519 - 0.95)/0.95 \cdot 30.15]] - 6.63 \approx 10.4 \text{ cm}$

d) MPV for slab

- $h_c$ (minimum critical thickness) = 10.6 cm
- rods diameter = 0.762 cm
- $M^2 = 30.41 \text{ cm}^2$
- $\delta = 6.63 \text{ cm}$
- $d = 135.57 \text{ g. }^{235}\text{U/liter}$

Substituting in eqs (3) and (4) for $\text{Keff.} = 1$

$K = 1 + [30.41 \text{ m}^2 / (10.6 + 2 \cdot 6.63)^2] \approx 1.51$

If $\text{Keff.} = 0.95$

$VMP$ for thickness $=\n= [\pi / [(1.51 - 0.95)/0.95 \cdot 30.41]^{1/2}] - 2 \times 6.83 \approx 8.9 \text{ cm}$
3. CRITERIA FOR INCREASING VMP

A way to increase MPV is limiting the degree of moderation of material. (relationship H/U). In other cases MPVs can be increased limiting one dimension in the geometry instead of considering it infinite.

3.1. INCREASED VMP AS FUNCTION OF RELATIONSHIP H/U FOR HOMOGENEOUS SYSTEMS

In the case of homogeneous system the reactivity of the material is determined by the relationship between H/U. If we are sure that in an homogeneous unit the relationship H/U has a certain maximum value, we can use it to increase the MPV. Of course this is only possible when that relationship H/U has already been determined by analysis and there is a moderated strict control. We have been calculating this taking as a starting the data in the Ref. 2, and representing the results in Fig. 1 and 2.

We have estimated, for an homogeneous system the maximum relationship H/U for a determined density UO₂, that is

\[ VF(UO₂) + VF(H₂O) = 1 \]

\[ VF = \text{Volume fraction} \]

For these relationships H/U we have calculated the values of a mass, volume, diameter of cylinder and thickness slab.
The ratio $H/U$ is calculated as follows

$$\frac{\text{Pmol} \text{ UO}_2}{\text{Pmol} \text{ U} \cdot \text{TD(UO}_2))} \cdot 2 \frac{1 - \frac{d(U)}{\text{Pmol} \text{ H}_2\text{O}}}{{\text{Pmol} \text{ U}}} \cdot \frac{d(U)}{\text{Pmol} \text{ U}}$$

$$= \frac{9.66}{2.74} \left[ - \frac{d(U)}{\text{Pmol} \text{ U}} - 1 \right]$$

Where we have substituted the following values:

\begin{align*}
VF &= \text{Volume fraction} \\
\frac{d(U)}{\text{Pmol} \text{ U}} &= \text{density of uranium} = g \text{ de } U/\text{cc} \\
\text{Pmol} \text{ U} &= 238 \\
\text{Pmol} \text{ UO}_2 &= 270 \\
\text{Pmol} \text{ H}_2\text{O} &= 18 \\
\text{TD(UO}_2) &= \text{theoretical density } UO_2 = 10.96 \text{ g/cc}
\end{align*}

Now we make the calculation according to the method described above. The results are represented in figures 1 and 2. We have obtained the values $K_1$ and $M_2$ for the different ratio $H/U$ from ref. 2.

a) **Safe batch (kg) as function as ratio $H/U$ (fig. 1)**

$$M = \frac{4\pi}{3} n^2 \left[ \frac{(K_1 - 0.95)/0.95 M_2^{1/2}}{M_2} \right]^3$$

b) **Volume (l) safe as function as ratio $H/U$ (fig. 1)**

Using eqs (1) and (4) we obtain

$$V = \frac{4\pi}{3} \pi \left[ \frac{(K_1 - 0.95)/0.95 M_2^{1/2}}{M_2} \right]^3$$
c) **Infinite cylinder safe radius** (cm) as function as ratio H/U (fig. 2)

Using eqs (2) and (4) we obtain

\[ R = \frac{2.405}{(K_{\infty} - 0.95)/0.95 \text{ M}^2} - \delta \]

\[ T = \frac{\pi}{(K_{\infty} - 0.95)/0.95 \text{ M}^2} - 2\delta \]

d) **Slab safe thickness** (cm) as function as ratio H/U (fig. 2)

e) **Maximum admissible H/U in moderated control area**

A good parameter for criticality control is the limitation of ratio H/U. We have fixed the maximum admissible relationship H/U in the fissil material. To determine this H/U we have estimated, using SCALE, the K- for different mixtures of UO₂ - H₂O until we have found an K-infinite = 1. This value is H/U = 0.46 and it can be taken as the maximum admissible H/U in moderated control area. Also we have calculated the radius of spheres full reflected by water with different densities of uranium for H/U = 0.46 and Keff = 0.95. The results are shown in fig 3. The moderation control must be completed by establishing a limit for the amount of H which due to a mistake can be incorporated in the material. To prevent this effect we suppose a small sphere UO₂ - H₂O with the most reactive composition (1) with a relationship H/U = 0.46 to immerse in an infinite medium We have been increasing the radius of inner sphere up to Keff = 0.95 (fig 4). This value is 16 liters. Its the limits of H₂O or equivalent moderator admitted in an area with moderation control.

(1) 2.27 g UO₂/cc and 0.793 g H₂O/cc
It's the limit of H₂O or equivalent moderator admitted in an area under moderation control.

3.2. INCREASING MPV FOR ACUMULATION RODS

This way of increasing the value of MPV can also be used with heterogeneous systems, although in this case the calculation is more complicated. The sequence of XSRNDRNPN-KENO IV of code SCALE has been used to increase the slab thickness for heterogeneous materials. In this way we have calculated the most reactive array rods. However in the installations most of the accumulation rods are compact. That means they are in contact one another and even in a case of flooding the systems are in the submoderated region. Using the most reactive type of rods of our plant and supposing them to be in contact forming a squarepitch, which is more reactive than the triangular pitch. We have done several calculations XSRNDRNPN-KENO IV for different thickness of an infinite slab of compact rods full reflected and flooded. The thickness that gives as a result a K-eff ≤ 0.95 (including the uncertainty) is of 23.3 cm, so for accumulations of this type the general MPV, 8.9 can be increased in more than the double.

3.3. INCREASING MPV FOR FINITE GEOMETRY

To increase MPVs we must bear in mind that the units are finite of limited dimensions. In fact the K-eff can be written.

\[
\text{K-eff} = \frac{1}{1 + M^2 B^2 G^2}
\]

Where K-eff and M² depend on the characteristics of the material and B²G² depends on geometry and slightly on the material.
a) Cylinder infinite

In the case of a finite cylinder with radius and length L: \( Bg^2 = Bg^2 (r, L, \delta) \). The \( Bg^2 \) of an infinite cylinder with the maximum permissible radius, \( r_m \), is \( Bg^2 (r_m, \infty, \delta) \) From the euation (1),

\[ Bg^2 (r, 1, \delta) = Bg^2 (r_m, \infty, \delta) \]

You can find in fig. 5 a function \( r=r(L) \)

b) Slab finite

In a similar way you can find the maximum permissible thickness of a slab a sides \( X \) and \( Y \),

\[ Bg^2 (X,Y,Z) = Bg^2 (X, \infty, \infty, \delta) \]

from Eq. (2)

\[
A = \frac{A}{B}
\]

\[ h_r = \frac{2 \delta}{A} \]

where,

\[ A = (h_r + 2 \delta)(X + 2 \delta)(Y + 2 \delta) \]

\[ B = \left\{ \frac{(X+2\delta)^2(Y+2\delta)^2}{(h_r + 2\delta)^2} - (h_r + 2\delta)^2(Y + 2\delta)^2 - (h_r + 2\delta)^2(X+2\delta)^2 \right\}^{1/2} \]

The results are shown in fig. 6 and 7.
4. SUMMARY

There is not a precise criteria to obtain the MPVs. We have chosen a method which makes $K_{-} = 0.95$ in the equation

$$K_{-} = \frac{K_{\text{eff}}}{1 + M^2 Bq^2}$$

The MPVs are usually used for very restrictive conditions which make more difficult their use. We have seen that taking into account finite units or limiting the relation $H/U$, less restrictive MPV can be defined, allowing a wider range usage.

The results are represented in table and figures.

5. REFERENCES


FIG. 1 VOLUME AND SAFE MASS AS A FUNCTION H/U ATOM RATIO

HOMOGENEOUS MIXTURE (UO₂ - H₂O)

ENRICHMENT 5 wt% ²³⁵U

VOLUME (L)

MASS (Kg)

LITERS

80-

70-

60-

50-

40-

30-

20-

10-

2 4 6 8 10

Kg

-500

-400

-300

-200

-100

40 50

H/U
FIG. 2  SLAB AND CYLINDER SAFE AS FUNCTION OF H/U ATOM RATIO
FIG. 3  RADIUS OF FULL REFLECTED
SPHERE AS FUNCTION OF DENSITY

RAZON H/U = 0.46
Kef (95/95) = 0.95
Keₙ as function of H₂O liters in most reactive inner sphere of an infinitum medium (UO₂, H/U = 0.46)

FIG. 4
FIG. 5. CYLINDER SAFE (Ref < 0.95) AS FUNCTION OF RADIUS AND HEIGHT. ENRICHMENT 5% WT $^{235}$U
FIG. 6 SLAB THICKNESS AS FUNCTION OF BASE DIMENSION FOR HOMOGENEOUS SYSTEM
FIG. 7 SLAB THICKNESS AS FUNCTION OF BASE
DIMENSION FOR HETEROGENEOUS SYSTEM