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Charged charmonium molecules

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We make use of a self-consistent quark-model based study of four-quark charmonium-like states to interpret recent charmonium experimental data. We conclude that there exists a $D^*\bar{D}^*$ meson-meson molecule with quantum numbers $(I^G)J^{PC} = (1^-)2^{++}$. Our study confirms the presence of charged charmonium-like resonances on the excited charmonium spectrum. We find support from recent experimental data by the Belle Collaboration [R. Mizuk *et al.* (Belle Collaboration), Phys. Rev. D **78**, 072004 (2008)]. Confirmation of the experimental data by the Belle Collaboration and the determination of the quantum numbers of the new structures would help in discriminating among different theoretical models and would give further support to the theoretical analysis of T. Fernández-Caramés, A. Valcarce, and J. Vijande [Phys. Rev. Lett. **103**, 222001 (2009)].

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Since 2003 several states have been discovered in the charmonium mass region. While in the conventional description of the charmonium spectrum in terms of quark-antiquark pairs some states are still missing, the number of experimental candidates reported up to now is larger than the empty spaces in the $c\bar{c}$ spectrum. This, together with other difficulties in explaining observed resonances as simple quark-antiquark pairs, triggered discussions on possible exotic interpretations: four-quark states either as compact tetraquarks or slightly bound meson-meson molecules.

Among the new armada of experimental observations, charged states have a unique feature: by construction, they cannot be accommodated in the conventional $c\bar{c}$ spectrum. Thus, their experimental confirmation would unavoidably drive the existence of hadronic molecules on the charmonium spectroscopy. Two different experimental findings show positive results on charged charmonium-like mesons. The first one is a peak that was observed by the Belle Collaboration [1] in the $\pi^\pm\Psi'$ invariant mass distribution near 4.43 GeV$/c^2$ in $B \to K\pi^\pm\Psi'$ decays. The discovery of charged charmonium-like states caused great interest and served as a motivation for searches of the analogous three-body decays of B mesons to other charmonium states. These analyses gave rise to the second positive observation, also reported by the Belle Collaboration [2], in exclusive $\bar{B}^0 \to K^-\pi^+\chi_{c1}$ decays. Resonance-like structures were observed in the $\pi^+\chi_{c1}$ invariant mass distribution around 4.1 GeV$/c^2$. From a Dalitz plot analysis representing the mass structures from Breit-Wigner resonance amplitudes, masses of about 4050 and 4250 MeV$/c^2$ were determined [2].

In the first case the *BABAR* Collaboration has presented its own analysis [3], performing a detailed study of the acceptance and possible reflections, concluding that no significant signal exists in the data. While the two experiments made different conclusions, the data themselves seem to be in reasonable agreement except for the lower available statistics of the *BABAR* experiment. The absence of the peak in the *BABAR* data is associated with different assumptions on the shape of the background. After these null results from *BABAR*, the Belle group performed a reanalysis of their data, taking detailed account of possible reflections in the $\pi\Psi'$ channel from the $K\pi$ channel [4]. Their results confirmed the basic conclusions of Belle's 2008 publication [1]. With respect to the second observation by the Belle Collaboration [2], the $M^2(K\pi)$ versus $M^2(\pi\chi_{c1})$ Dalitz plot shows vertical bands of events corresponding to $K^*(890) \to K\pi$ and $K_2^*(1430) \to K\pi$, plus a broad horizontal band near $M^2(\pi\chi_{c1}) \simeq 17.5$ GeV$^2/c^4$, indicating a possible resonance in the $\pi^+\chi_{c1}$ channel. The fit with a single resonance in the $Z \to \pi\chi_{c1}$ channel is favored over a fit with only K^* resonances and no Z by more than 10σ. Moreover, a fit with two resonances in the $\pi\chi_{c1}$ channel is favored over the fit with only one Z resonance by 5.7σ. Since these results were observed in the $\pi^+\chi_{c1}$ channel, the only quantum numbers that are known about these structures are $I^G = 1^-$. In this case, there has not been a cross-check of the results by a different collaboration, which would be highly desirable. In particular, the analogous analysis of the Dalitz plot for the data of the *BABAR* Collaboration could help in clarifying the, to some extent, ambiguous experimental conclusion.

The idea of unconventional quark structures is quite old [5], and despite decades of progress, no exotic meson has been conclusively identified. The structures observed in charged channels in the charmonium region should be of primary interest because they would make manifest the existence of four-quark contributions to the meson spectroscopy. This challenging conclusion seems rather difficult to obtain in other cases, like in the case of scalar mesons or uncharged charmonium resonances, where experimental data will hardly exclude a theoretical model [6]. Besides the explanation of the $X(3872)$ given in Ref. [7], we predicted the existence of an attractive $D\overline{D}$ charged channel, $(I^G)J^{PC} = (1^-)2^{++}$. We revisit our analysis in view of the experimental support of Ref. [2]. The idea behind our analysis was that charmonium spectroscopy could be rather simple below the threshold production of charmed mesons, but much more complex above it. In particular, the existence of $(c\bar{c})(n\bar{n})$ thresholds, with an attractive interaction between the corresponding $D\bar{D}$ two-meson system, could be an important spectroscopic ingredient, which has been called *the unquenching of the naive quark model* [8]. In this way, hidden-charm four-quark states could explain the overpopulation of quark-antiquark theoretical states and, at the same time, the existence of unconventional structures like charged charmonium-like mesons. Thus, the new experimental data offer exciting new insights into the subtleties of the strong interaction.

To study the possible contribution of charged multiquark configurations to the charmonium spectroscopy, we have solved the Lippmann-Schwinger equation for negative energies using the Fredholm determinant. This method permitted us to obtain robust predictions even for zero-energy bound states, and gave us information about attractive channels that may contain a meson-meson molecule [7]. In order to account for all basis states [9] we allow for the coupling to charmonium-light two-meson channels. Thus, we consider the system of two mesons M_1 and $\bar{M}_2$ ($M_i = D, D^*$) in a relative S state interacting through a potential V that contains a tensor force. Then, in general, there is a coupling to the $M_1\bar{M}_2 D$ wave. Moreover, the two D-meson system can couple to a charmonium-light two-meson state through quark rearrangement; see Fig. 1, where as an example $D^*\bar{D}^*$ is coupled to $J/\Psi\rho$. Thus, if we denote the two D-meson system as channel D and the charmonium-light system as channel Ψ, the Lippmann-Schwinger equation for $M_1\bar{M}_2$ scattering becomes

$$t^{\ell_\alpha s_\alpha,\ell_\beta s_\beta}_{\alpha\beta;ji}(p_\alpha, p_\beta; E) = V^{\ell_\alpha s_\alpha,\ell_\beta s_\beta}_{\alpha\beta;ji}(p_\alpha, p_\beta) + \sum_{\gamma=D,\Psi}\sum_{\ell_\gamma=0,2}\int_0^\infty p_\gamma^2 dp_\gamma V^{\ell_\alpha s_\alpha,\ell_\gamma s_\gamma}_{\alpha\gamma;ji}(p_\alpha, p_\gamma) G_\gamma \times (E; p_\gamma) t^{\ell_\gamma s_\gamma,\ell_\beta s_\beta}_{\gamma\beta;ji}(p_\gamma, p_\beta; E), \qquad \alpha, \beta = D, \Psi, \tag{1}$$

where t is the two-body scattering amplitude; j, i, and E are the angular momentum, isospin, and energy of the system; $\ell_\alpha s_\alpha$, $\ell_\gamma s_\gamma$, and $\ell_\beta s_\beta$ are the initial, intermediate, and final orbital angular momentum and spin, respectively, and p_γ is the relative momentum of the two-body system γ. The propagators $G_\gamma(E; p_\gamma)$ are given by

$$G_\gamma(E; p_\gamma) = \frac{2\mu_\gamma}{k_\gamma^2 - p_\gamma^2 + i\epsilon}, \tag{2}$$

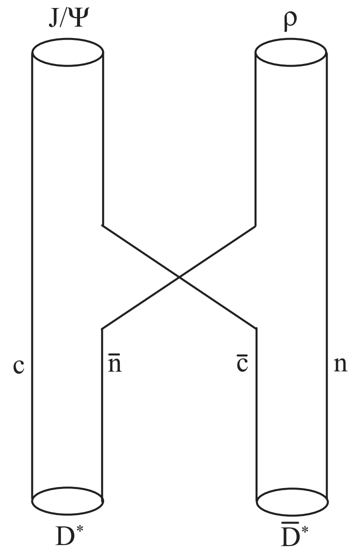

FIG. 1. Quark rearrangement diagram generating transitions $D^*\bar{D}^* \leftrightarrow J/\Psi\rho$.

with

$$E = \frac{k_\gamma^2}{2\mu_\gamma}, \tag{3}$$

where μ_γ is the reduced mass of the two-body system γ. For bound-state problems $E < 0$ so that the singularity of the propagator is never touched and we can forget the $i\epsilon$ in the denominator. If we make the change of variables

$$p_\gamma = b\frac{1 + x_\gamma}{1 - x_\gamma}, \tag{4}$$

where b is a scale parameter, and the same for p_α and p_β, we can write Eq. (1) as

$$t^{\ell_\alpha s_\alpha,\ell_\beta s_\beta}_{\alpha\beta;ji}(x_\alpha, x_\beta; E) = V^{\ell_\alpha s_\alpha,\ell_\beta s_\beta}_{\alpha\beta;ji}(x_\alpha, x_\beta) + \sum_{\gamma=D,\Psi}\sum_{\ell_\gamma=0,2}\int_{-1}^1 b^2\left(\frac{1 + x_\gamma}{1 - x_\gamma}\right)^2 \frac{2b}{(1 - x_\gamma)^2} dx_\gamma \times V^{\ell_\alpha s_\alpha,\ell_\gamma s_\gamma}_{\alpha\gamma;ji}(x_\alpha, x_\gamma) G_\gamma(E; p_\gamma) t^{\ell_\gamma s_\gamma,\ell_\beta s_\beta}_{\gamma\beta;ji}(x_\gamma, x_\beta; E). \tag{5}$$

We solve this equation by replacing the integral from -1 to 1 by a Gauss-Legendre quadrature, which results in the set of linear equations

$$\sum_{\gamma=D,\Psi}\sum_{\ell_\gamma=0,2}\sum_{m=1}^N M^{n\ell_\alpha s_\alpha,m\ell_\gamma s_\gamma}_{\alpha\gamma;ji}(E) t^{\ell_\gamma s_\gamma,\ell_\beta s_\beta}_{\gamma\beta;ji}(x_m, x_k; E) = V^{\ell_\alpha s_\alpha,\ell_\beta s_\beta}_{\alpha\beta;ji}(x_n, x_k), \tag{6}$$

with

$$M_{\alpha\gamma;ji}^{n\ell_\alpha s_\alpha,m\ell_\gamma s_\gamma}(E) = \delta_{nm}\delta_{\ell_\alpha\ell_\gamma}\delta_{s_\alpha s_\gamma} - w_m b^2\left(\frac{1+x_m}{1-x_m}\right)^2 \times \frac{2b}{(1-x_m)^2} V_{\alpha\gamma;ji}^{\ell_\alpha s_\alpha,\ell_\gamma s_\gamma}(x_n, x_m) G_\gamma(E; p_{\gamma m}), \tag{7}$$

and where w_m and x_m are the weights and abscissas of the Gauss-Legendre quadrature [10], while $p_{\gamma m}$ is obtained by putting $x_\gamma = x_m$ in Eq. (4). If a bound state exists at an energy E_B, the determinant of the matrix $M_{\alpha\gamma;ji}^{n\ell_\alpha s_\alpha,m\ell_\gamma s_\gamma}(E_B)$ vanishes, i.e., $|M_{\alpha\beta;ji}(E_B)| = 0$. We took the scale parameter b of Eq. (4) as $b = 3\ \mathrm{fm}^{-1}$ and used a Gauss-Legendre quadrature with $N = 20$ points.

We make use of the same interacting potential and parameters, containing universal one-gluon exchange, confinement, and a chiral potential [11], to study the two- and four-quark systems. For this purpose we have solved the coupled channel problem of the $D\bar{D}$, $D\bar{D}^*$, and $D^*\bar{D}^*$. In all cases we have included the coupling to the relevant $(c\bar{c})(n\bar{n})$ channels (denoted generically as $J/\Psi\rho$). The results for the uncharged $I = 0$ channels have been discussed in detail in Ref. [7]. In this work we want to focus thoroughly on the study of all possible charged channels, $I = 1$, considering relative orbital angular momentum up to $L = 2$ and all possible total spins: $S = 0$, 1, and 2. No attractive isospin-one channels have been observed, either in the $D\bar{D}$ or in the $D\bar{D}^*$ systems. The single channel calculation, using only $D\bar{D}$ or $D\bar{D}^*$ channels, is never attractive enough to generate a bound state or a resonance, and the coupling to the corresponding $\eta_c\pi$ or $J/\Psi\pi$ channel diminishes the attraction due to the large mass difference with the threshold. Only the $D^*\bar{D}^*$ shows an attractive channel, the $(I^G)J^{PC} = (1^-)2^{++}$. Amazingly, these are the isospin and G-parity quantum numbers of the experimental bumps reported in Ref. [2], and therefore our results for the spin, parity, and C parity may be considered as a prediction. Let us emphasize the general study we have done, not looking for any particular resonance, but trying to sort the different $(I^G)J^{PC}$ channels in terms of their attractive or repulsive character.

It is easy to explain the reason for such a close-to-bind situation with the above quantum numbers. As it occurred in the case of the $X(3872)$, $J^{PC} = 2^{++}$ can be reached from two D^* mesons without explicit relative orbital angular momentum. Because of its color content, the one-gluon exchange participates only through quark-exchange diagrams [12], giving a small short-ranged contribution. It is the pseudoscalar term of the chiral potential that differentiates among the several total spin channels. Thus, the expectation value of the $\vec{\sigma}_i \cdot \vec{\sigma}_j$ operator contributing in the pseudoscalar interaction is 1 for $S = 2$, -1 for $S = 1$, and -2 for $S = 0$, making the spin-2 channel the most attractive. The coupling to the charmonium-light two-meson system takes the remaining role by increasing the attraction; see Fig. 2. The close-to-bind situation is illustrated by the dashed-dotted line, where we have multiplied the potential by a fudge factor of 1.2, generating a bound state just below threshold. With a P-wave resonance in the $\pi^+\chi_{c1}$ channel, a width of the order of 100–200 MeV is expected [13]. The resonance should also be seen in the $J/\Psi\pi^+\pi^0$ channel, and through its electromagnetic decay to $J/\Psi\pi^+\gamma$. The diminishing of the attractive character when decreasing the total spin is illustrated in Fig. 3. The $D^*\bar{D}^*$ spin-1 channel, with quantum numbers $(I^G)J^{PC} = (1^+)1^{+-}$, is less attractive due, first, to the expectation value of $\vec{\sigma}_i \cdot \vec{\sigma}_j$, and second, to the coupling to the $J/\Psi\pi$ channel.

We have investigated within our model the possible existence of a second peak with the same quantum numbers close in energy. Such a situation could resemble what has been pointed out by D. V. Bugg [14]. In the light meson spectra, in the mass range 1900 MeV$/c^2$ upwards there are *towers* of resonances that are not exactly degenerate states. The total $\bar{p}p$ cross section shows small bumps at these masses, although only at the few percent level. Looking carefully, one observes a regular progression of masses with orbital angular momentum; on average, there is 40 MeV$/c^2$ mass difference for each unit of orbital angular momentum. All these states follow straight-line Regge trajectories quite well [6]. Looking for the same possibility in the charmonium exotic states, we have studied in detail the $(1^-)2^{++}$ quantum numbers obtained from the D wave instead of the S wave. A similar situation to the one observed for light mesons may give rise to a second bump, on average 80 MeV$/c^2$ above the first one. We have, first of all, used our machinery based on the hyperspherical harmonic formalism [15], looking for the

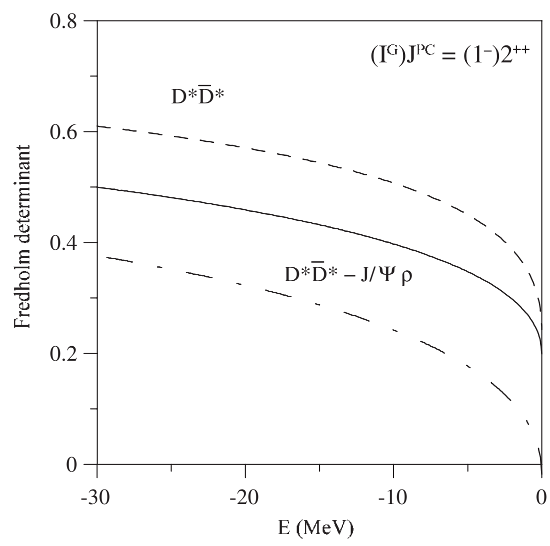

FIG. 2. Fredholm determinant for the $(I^G)J^{PC} = (1^-)2^{++}\,D^*\bar{D}^*$ system. Solid (dashed) line: results with (without) coupling to the $J/\Psi\rho$ channel. The dashed-dotted line stands for the coupled channel calculation with a fudge multiplicative factor in the interaction, as explained in the text.

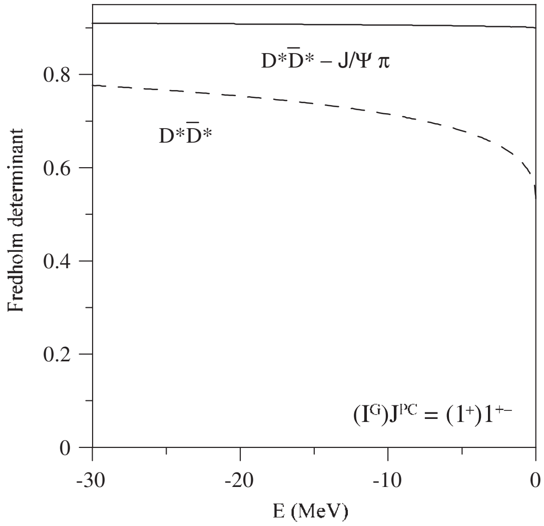

FIG. 3. Fredholm determinant for the $(I^G)J^{PC} = (1^+)1^{+-} D^*\bar{D}^*$ system. Solid (dashed) line: results with (without) coupling to the $J/\Psi\pi$ channel.

existence of possible resonances and/or bound states. We present in Table I the results we have obtained. As can be seen, the D wave evolves asymptotically to the same energy of the S wave. Besides, the radius increases linearly, pointing clearly to a well-separated two-meson state; therefore, it does not show any possibility of having an excited D-wave state. For completeness, we have solved the $D^*\bar{D}^* L = 2$ coupled channel problem with and without coupling to the corresponding charmonium-light two-meson system. The Fredholm determinant for the D wave is shown in Fig. 4. As can be seen, there is no attraction, discarding the existence of an observable excited state with these quantum numbers.

There are other studies in the literature devoted to these particular charged states. Studies based on QCD sum rules [16], without drawing a definite conclusion, prefer the existence of a $(I^G)J^P = (1^-)1^- D_1\bar{D}$ molecular state compatible with the bump around 4250 MeV$/c^2$ reported in Ref. [2] and speculate about the $Z(4050)$ being a threshold effect associated with $\eta_c''(3^1S_0)$. However, Refs. [17], based also in QCD sum rules but using different interpolating fields for the $J^P = 1^-$ channel, prefer a scalar tetraquark $J^P = 0^+$ for the $Z(4250)$, obtaining masses of 5.12 GeV$/c^2$ for the vector $J^P = 1^-$ one. The meson-exchange model of Ref. [18], which combines the heavy quark symmetry and chiral symmetry, concludes that the $Z(4250)$ cannot be a meson-meson molecule, while it finds a strong attraction in the $J^P = 0^+$ $D^*\bar{D}^*$ channel compatible with the $Z(4050)$. Reference [19], also based on meson-exchange models, concluded that the interpretation of the $Z(4050)$ as a $D^*\bar{D}^*$ molecule is not favored. A relativistic quark-antidiquark approach does not find a tetraquark candidate for the $Z(4050)$ and suggests that the $Z(4250)$ could be a $J^P = 0^-$ or 1^- hidden-charm tetraquark [20].

Regarding the $Z(4430)$ [1], we cannot give a definitive conclusion. The obvious nonexistence of orbital excited states in our description would rule out its existence. However, its vicinity to a threshold containing an orbital D-meson excited state has suggested the further study of the interaction between radial and orbital excitations of D mesons, which will allow us, in the near future, to achieve some conclusion regarding states with masses above 4.3 GeV$/c^2$, and, in particular, about the $Z(4430)$.

As a summary, we have studied in detail the possible existence of charged charmonium molecules above the threshold for the production of D mesons. We have analyzed all possible quantum numbers within the same model, and we have used two different approaches and numerical methods, giving confidence to our results. Finally, let us note that our study was not intended to look for any particular experimental resonance, but pretended to sort all quantum numbers of the two D-meson system regarding their attractive or repulsive character. The first remarkable conclusion, already pointed out in Ref. [7], is that no deeply bound states can be expected for charged charmonium-like states. Our results point to

TABLE I. Hyperspherical harmonic results for the $(I^G)J^{PC} = (1^-)2^{++} c\bar{c}n\bar{n}$ quantum numbers as a function of the grand angular momentum K. Energies are in MeV and radii are in fm.

K	$E_{L=0}$	$E_{L=2}$	$\Delta(E_{L=2} - E_{L=0})$	$(\langle r^2\rangle)^{1/2}_{L=0}$	$(\langle r^2\rangle)^{1/2}_{L=2}$
2	4057.4	4498.9	441.5	0.378	0.467
4	3999.1	4279.3	280.2	0.417	0.520
6	3967.7	4170.3	202.6	0.457	0.581
8	3948.1	4104.6	156.5	0.497	0.639
10	3934.8	4060.8	126.0	0.537	0.696
12	3925.2	4029.6	104.4	0.578	0.753
14	3918.1	4006.5	88.4	0.617	0.808
16	3912.6	3988.8	76.2	0.656	0.863
18	3908.2	3974.8	66.6	0.695	0.917
20	3904.7	3963.5	58.8	0.731	0.970

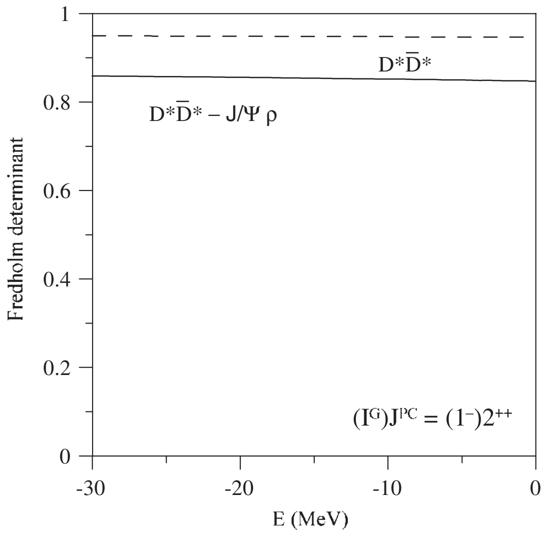

FIG. 4. Fredholm determinant for the $(I^G)J^{PC} = (1^-)2^{++} D^*\bar{D}^*$ system with $L = 2$. Solid (dashed) line: results with (without) coupling to the $J/\Psi\rho$ channel.

the existence of a $(I^G)J^{PC} = (1^-)2^{++}$ charged resonance just above the $D^*\bar{D}^*$ threshold, and therefore with a mass of the order of 4030 MeV$/c^2$. Such a prediction finds experimental support in the results of Ref. [2]. The isospin and G parity can be extracted from the experimental analysis, with the spin, parity, and C parity derived from our theoretical calculation. We do not find any evidence for the existence of a second charged state in the same mass region. As the experimental data of Ref. [2] are compatible with the existence of a single Z state, this is the option preferred by our theoretical model.

We encourage the *BABAR* Collaboration to perform an analogous analysis of the Dalitz plot to clarify the experimental situation. A positive finding of a charmonium-like charged state will definitively help in discriminating between different theoretical models [the diquark-antidiquark hypothesis predicts the existence of charged partners for the $X(3872)$] for the hadron spectroscopy and will open an unknown and interesting new spectroscopic era.

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