Strangeness $-2$ two-baryon systems

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1. Introduction

The knowledge of the strangeness $S = -2$ two-baryon interactions has become an important issue for theoretical and experimental studies of the strangeness nuclear physics. Moreover, this is an important piece of a more fundamental problem, the description of the interaction of the different members of the baryon octet in a unified way. The $S N - \Lambda \Lambda$ interaction accounts for the existence of doubly strange hypernuclei, which is a gateway to strange hadronic matter. Strangeness $-2$ baryon–baryon interactions also account for a possible six-quark $H$-dibaryon, which has yet to be experimentally observed. Its knowledge is naturally invaluable for the quantitative predictions for various aspects of neutron star matter.

There has been an steady progress towards the $S = -2$ baryon–baryon interaction. The $\Lambda N$ interaction is pretty much understood based on the experimental data of $\Lambda$ hypernuclei. There is also some progress made on the $\Sigma N$ interaction. However, the experimental knowledge on the $\Sigma N$ and hyperon–hyperon ($YY$) interactions is quite poor. The only information available came from doubly strange hypernuclei, suggesting that the $^1S_0 \Lambda \Lambda$ interaction should be moderately attractive. An upper limit of $B_{\Lambda \Lambda} = 7.25 \pm 0.19$ MeV has been deduced for an hypothetical $H$-dibaryon (a lower limit of 2223.7 MeV/$c^2$ for its mass) from the so-called Nagara event [1] at a 90% confidence level. The KEK-E176/E373 hybrid emulsion experiments observed other events correspond-

We derive strangeness $-2$ baryon–baryon interactions from a chiral constituent quark model including the full set of scalar mesons. The model has been tuned in the strangeness $0$ and $-1$ two-baryon systems, providing parameter free predictions for the strangeness $-2$ case. We calculate elastic and inelastic $N \Xi$ and $\Lambda \Lambda$ cross sections which are consistent with the existing experimental data. We also calculate the two-body scattering lengths for the different spin–isospin channels.
giving a nice description of the hyperon–nucleon elastic and inelastic cross sections, and valuable predictions for the strangeness \(-1\) three-baryon systems: \(\Lambda NN\) [11].

In this work, we will apply the same model to derive the strangeness \(-2\) baryon–baryon interactions: \(\Lambda\Lambda, \Lambda \Sigma, \Sigma \Sigma\) and \(N \Xi\). We will use these two-body interactions to calculate two-body elastic and inelastic scattering cross sections and we will compare to experimental data and other theoretical models. We will also calculate the two-body scattering lengths of the different spin–isospin channels to compare with other theoretical models. The structure of the Letter is the following. In the next section we will resume the basic aspects of the two-body interactions and we will present the integral equations for the different two-body systems. In Section 3 we present our results compared to other models and the available experimental data. Finally, in Section 4 we summarize our main conclusions.

2. Formalism

2.1. The strangeness \(-2\) baryon–baryon potential

The baryon–baryon interactions involved in the study of the coupled \(\Lambda\Lambda – N \Xi – \Lambda \Sigma – \Sigma \Sigma\) system are obtained from the chiral constituent quark model [9]. In this model baryons are described as clusters of three interacting massive (constituent) quarks, the mass coming from the spontaneous breaking of chiral symmetry. The first ingredient of the quark–quark interaction is a confining potential (CON). Perturbative aspects of QCD are taken into account by means of a one-gluon potential (OGE). Spontaneous breaking of chiral symmetry gives rise to boson exchanges between quarks. In particular, there appear pseudoscalar boson exchanges and their corresponding scalar partners [11]. Thus, the quark–quark interaction will read:

\[
V_{qq}(\vec{r}_{ij}) = V_{\text{CON}}(\vec{r}_{ij}) + V_{\text{OGE}}(\vec{r}_{ij}) + V_{\chi}(\vec{r}_{ij}) + V_{S}(\vec{r}_{ij}),
\]

where the \(i\) and \(j\) indices are associated with \(i\) and \(j\) quarks respectively, and \(\vec{r}_{ij}\) stands for the interquark distance. \(V_{\chi}\) denotes the pseudoscalar meson-exchange potential and \(V_{S}\) stands for the scalar meson-exchange potential described in Ref. [11]. Explicit expressions of all the interacting potentials and a more detailed discussion of the model can be found in Refs. [10,11]. In order to derive the local \(B_1B_2 \to B_1B_2\) potentials from the basic quq interaction defined above we use a Born–Oppenheimer approximation. Explicitly, the potential is calculated as follows,

\[
V_{B_1B_2(LST)} \to B_1B_2(LST) (R) = \xi_{LST}^{\Lambda\Lambda}(R) - \xi_{LST}^{\Lambda\Lambda}(\infty),
\]

where

\[
\xi_{LST}^{\Lambda\Lambda}(R) = \frac{\left\langle \psi_{LST}^{B_1B_2} (\vec{R}) \right| \sum_{i<j} V_{qq}(\vec{r}_{ij}) \left| \psi_{LST}^{B_1B_2} (\vec{R}) \right\rangle}{\left\langle \psi_{LST}^{B_1B_2} (\vec{R}) \right| \psi_{LST}^{B_1B_2} (\vec{R}) \right\rangle}.
\]

In the last expression the quark coordinates are integrated out keeping \(R\) fixed, the resulting interaction being a function of the \(B_1B_2\) relative distance. The wave function \(\psi_{LST}^{B_1B_2}(\vec{R})\) for the two-baryon system is discussed in detail in Ref. [9].

2.2. Integral equations for the two-body systems

If we consider the system of two baryons \(B_1\) and \(B_2\) with strangeness \(-2\), \(N \Xi\) and \(Y_1Y_2\) \((Y_1 = \Sigma, \Lambda)\) in a relative \(S\)-state interacting through a potential \(V\) that contains a tensor force, then there is a coupling to the \(B_1B_2\) \(D\)-wave so that the Lippmann–Schwinger equation for the spin singlet channels of Table 1 is of the form

\[
t_{\alpha \beta ; jj}(p, p'; W) = V_{\alpha \beta ; jj}(p, p') + \sum_{\gamma} \int_0^\infty p d\epsilon V_{\alpha \gamma ; jj}(p, \epsilon) \frac{2\mu_{\gamma}}{k_{\gamma}^2 - p^2 + i\epsilon} t_{\gamma \beta ; jj}(p', p'; W),
\]

where \(\mu_{\gamma}\) the reduced mass of the system and the on-shell momenta \(k_{\gamma}\) are defined as

\[
W = \sqrt{m_{1\gamma}^2 + k_{\gamma}^2} + \sqrt{m_{2\gamma}^2 + k_{\gamma}^2},
\]

where \(m_1\) and \(m_2\) are the masses of the particles of channel \(\gamma\).

2.3. Scattering cross sections

We now turn to the available low-energy data on the \(N \Xi\) scattering. There is only a small amount of data corresponding to the total cross sections for \(\Xi^{-}p \to \Xi^{-}p, \Xi^{-}p \to \Xi^{0}n,\) and \(\Xi^{-}p \to \Lambda \Lambda\) reactions. In the case of processes of the type \(\Xi^{-}N \to \Xi^{-}N\) the amplitudes obtained from Eqs. (4) and (5) are related to the cross section for a given isospin state through,

\[
\sigma_i = \pi^3 \mu_{\Xi N}^2 \left(3 |t_{00}^{\Lambda \Lambda = N \Xi; 1i}|^2 + |t_{00}^{\Lambda \Lambda = N \Xi; 0i}|^2 \right).
\]

From the isospin cross sections the physical channels are determined through,

\[
\sigma_{\Xi^{-}p \to \Xi^{-}p} = \frac{1}{4} \sigma_{i=0} + \frac{1}{4} \sigma_{i=0} + \frac{1}{2} \sqrt{\sigma_{i=0} \sigma_{i=1}},
\]

\[
\sigma_{\Xi^{-}p \to \Xi^{0}n} = \frac{1}{4} \sigma_{i=0} + \frac{1}{4} \sigma_{i=0} - \frac{1}{2} \sqrt{\sigma_{i=0} \sigma_{i=1}}.
\]

In the case of the process \(\Xi^{-}N \to \Lambda \Lambda\) it is necessary to include also the transition with \(i = 2\) in the \(\Lambda \Lambda\) channel. Thus, in that case the cross section for isospin \(i = 0\) is

\[
\sigma_0 = \pi^3 \mu_{\Xi N} \mu_{\Lambda \Lambda} \frac{k_{\Lambda \Lambda}}{k_{N \Xi}^2} \left( |t_{00}^{\Lambda \Lambda = N \Xi; 00}|^2 + 3 |t_{00}^{\Lambda \Lambda = N \Xi; 10}|^2 + 3 |t_{00}^{\Lambda \Lambda = N \Xi; 10}|^2 \right),
\]

Table 1

<table>
<thead>
<tr>
<th>(i)</th>
<th>(j = 0)</th>
<th>(j = 1)</th>
<th>(j = 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i = 0)</td>
<td>(\Lambda \Lambda – N \Xi – \Sigma \Sigma)</td>
<td>(N \Xi – \Lambda \Lambda)</td>
<td>(\Sigma \Sigma)</td>
</tr>
<tr>
<td>(i = 1)</td>
<td>(N \Xi)</td>
<td>(N \Xi – \Lambda \Lambda – \Sigma \Sigma)</td>
<td></td>
</tr>
<tr>
<td>(i = 2)</td>
<td></td>
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\(t_{\alpha \beta; jj}(p, p'; W)\) gives a nice description of the hyperon–nucleon elastic and inelastic cross sections, and valuable predictions for the strangeness \(-1\) three-baryon systems: \(\Lambda NN\) [11].

3. Results

Our results for the scattering cross sections are depicted by the solid lines in Figs. 1–4, compared to the available experimental data. The scattering lengths of the different spin–isospin channels are given in Table 2 compared to other theoretical models when available.

We show in Fig. 1 the $\Xi^- p \rightarrow \Sigma^0 n$ cross section, in mb, as a function of the laboratory momentum, in GeV/c. The experimental data is taken from Ref. [5]. A more recent experimental analysis [5] for the low energy $\Xi^- p \rightarrow \Sigma^0 n$ reaction in emulsion plates yielded 12.7 ± 3.1 mb in the total inelastic cross section in the momentum range 0.4–0.6 GeV/c [15], consistent with the results of Ref. [14]. The experimental results for $\Xi^- p$ inelastic scattering of Refs. [14,15] involve both $\Xi^- p \rightarrow \Lambda \Lambda$ and $\Xi^- p \rightarrow \Xi^0 n [5]$, what combined with the value for $\Xi^- p \rightarrow \Lambda \Lambda$ of Ref. [5] allows to obtain an estimate of the inelastic cross section $\Xi^- p \rightarrow \Xi^0 n$. Finally in Fig. 4 we present our prediction for the $\Lambda \Lambda$ scattering cross section. In this last case there are no experimental data.

As can be seen our results agree with the experimental data for the elastic and inelastic $\Xi N$ cross sections. The small bumps in the cross sections correspond to the opening of inelastic channels. As explained above, the interacting model is taken for grant from Ref. [11], where three body systems with strangeness −1 were studied, involving therefore two-body subsystems with strangeness 0 and −1. The agreement with the experimental data gives support to the dynamical model used and make our predictions valuable for forthcoming experiments. We would like to emphasize the agreement of our results with the $\Xi^- p \rightarrow \Lambda \Lambda$ conversion cross section. This reaction is of particular importance in assessing the
cepted that the ΛΛ since the observation of the Nagara event[1] it is generally ac-
prisition in our model would be a concrete calculation of

stability of Ξ− quasi-particle states in nuclei. Our results are close
to the estimations of the Nijmegen-M dodel [20], Ref. [16] predicts
σ(Σ− p → Ξ−0) ∼ 15 mb at plab = 0.5 GeV/c. Ref. [14] reported
14 mb for the inelastic scattering involving both Ξ− p → ΛΛ and
Σ− p → Ξ−0. We found a smaller value of approximately 6 mb
for both inelastic channels, in close agreement to experiment. In
the case of the elastic ΛΛ cross section there are no experimental
data. Our predictions are rather similar to Refs. [16,17]. We defi-
nitely need more experimental data with high statistics. This will
help us in discriminating among the different dynamical models. A
theoretical evaluation of the in-medium cross sections would also
be necessary.

The scattering lengths for the different spin-isospin channels are
given in Table 2. These parameters are complex for the NΣ
(i, j) = (0, 0) since the inelastic ΛΛ channel is open, for the ΛΣ
isospin 1 channel due to the opening of the NΣ channel, and for
the ΣΣ i = 1 and 0 since the ΛΛ and NΣ channels are open in the first case, and the NΣ and ΛΣ are open in the second
(see Table 1). There is no direct comparison between our results
and those of Ref. [16]. Although both are quark-model based
results, Ref. [16] used an old-fashioned quark-model interacting
potential. For example, they use a huge strong coupling constant,
αS = 1.9759, and they consider the contribution of vector mesons
what could give rise to double counting problems [21]. As ex-
plicitly written in Ref. [16], the parameters are effective in their
approach and has very little to do with QCD. As mentioned above,
since the observation of the Nagara event [1] it is generally ac-
cepted that the ΛΛ interaction is only moderately attractive. Our
result for the 1Σ0 ΛΛ scattering length is compatible with such
event. A rough estimate of BΛΛ(ΣΛΛ ≈ (hc)2/2μΛΛ(aΛΛ)2) drives
a value of 5.41 MeV, below the upper limit extracted from the
Nagara event, BΛΛ = 7.25 ± 0.19 MeV. Moreover, although from
the ΛΛ scattering length alone one cannot draw any conclusion on
the magnitude of the two-Λ separation energy, recent esti-
mates [22] have reproduced the two-Λ separation energy, defined as
ΔBΛΛ = BΛΛ(ΣΣ(2)He) − 2BΛΛ(2Σ(0)He), with scattering lengths of
−1.32 fm. The only reliable way to determine the two-Λ sepa-
ration energy in our model would be a concrete calculation of
doubly-strange hypernuclei. This has not been done so far, and it
is not clear that such calculation might help to further constrain
the potential model.

4. Summary and outlook

In this Letter we have presented the first results for the dou-
bly strange NΣ and Y1Y2 interactions (Yi = Σ, Λ) obtained with
a constituent quark model approach designed to study the non-
strange hadron phenomenology. The interaction incorporates long-
ranged meson-exchange contributions and the short-range dynam-
ics generated by the one-gluon exchange and quark antisymmetry
contributions.

We showed that the CCQM predictions are consistent with the
recently obtained doubly strange elastic and inelastic scattering
cross sections. In particular, our results are compatible with the
Σ− p → ΛΛ conversion cross section, important in assessing the
stability of Ξ− quasi-particle states in nuclei. Furthermore a mo-
derately attractive ΛΛ interaction arises. The presently available
scattering data are, however, not sufficient to draw definitive con-
clusions about the model, but forthcoming experimental data of the
observables reported will help in testing different theoretical
model predictions.

It is expected that in the coming years better-quality data on the
fundamental NΣ and YY interactions as well as much informa-
tion about the physics of hypernuclei will become available at the
new facilities J-PARC (Tokai, Japan) and FAIR (Darmstadt, Germany).
The CCQM developed here can then be used to analyze these up-
coming data in a model-independent way.

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