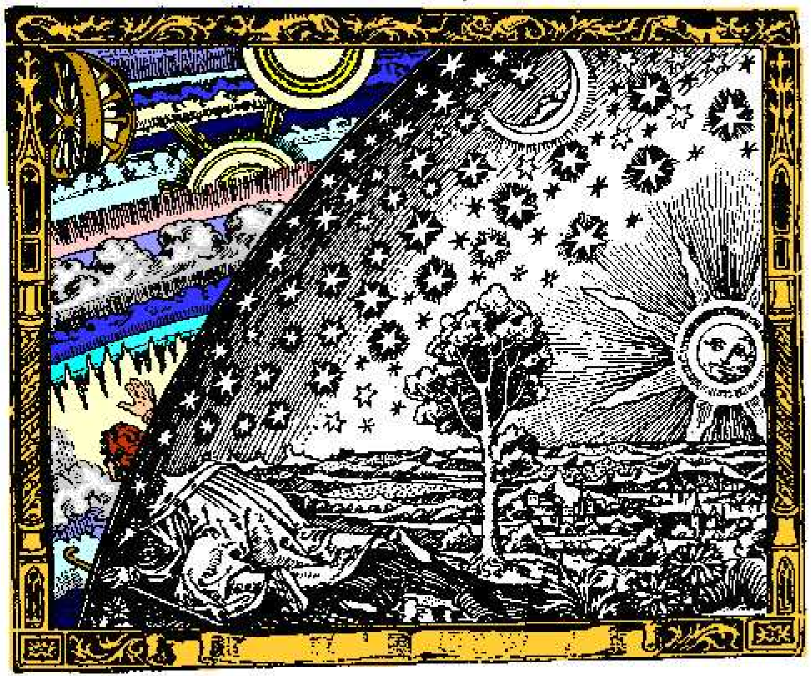
Cosmic Microwave Background and Large Scale Structure.



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Indice.

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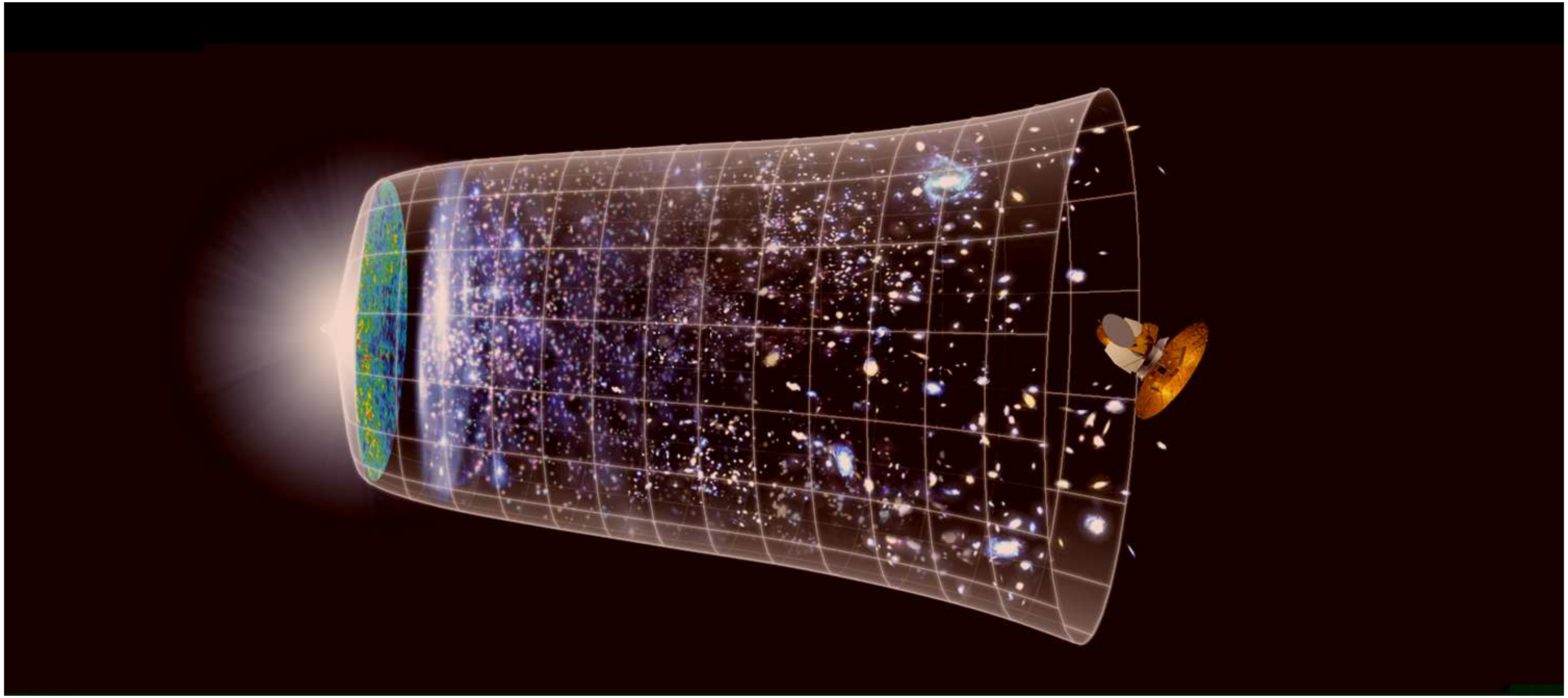
Introduction.

Timeline: from Light to the Dark Ages.



$\leftarrow l_{pl} = (hG/c^3)^{1/2} = 4 \cdot 10^{-33}\mathrm{cm}$
$t_{pl} = l_{pl}/c = 1.3 \cdot 10^{-43}\mathrm{s}$

10^{30}K — 10^{-40}s

$m_{pl} = (hc/G)^{1/2} = 5.5 \cdot 10^{-5}\mathrm{g}$

← Inflation.

← Symmetry-breaking: Gravity and strong-electroweak force.

10^{25}K — 10^{-30}s

10^{20}K — 10^{-20}s

← Symmetry-breaking: electro-weak and strong forces.

10^{15}K — 10^{-10}s

← Baryogenesis. Formation of protons and neutrons from quarks.

10^{10}K — 1s

← e^+, e^- - annihilation, Neutrino decoupling. Helium synthesis (25 - 30%).

			Matter dominated Universe.
10^5K	10^{10}s	←	Structure formation in the Universe.
		←	Hydrogen recombines. CMB Radiation propagates freely.
		←	Formation of the Sun.
10^{-2}K	10^{20}s	←	End of Stars. Entering the Dark Ages.
		←	End of Galaxies.
	10^{30}s		
		←	Proton decay (in e^+, γ, ν).
	10^{40}s		

10^{60}s Years. Black hole evaporation.

Parameters.

- ◇ Cosmological densities: Ω_Λ, $\Omega_m = \Omega_b + \Omega_{cdm} + \Omega_\nu$, Ω_γ, Ω_k
- ◇ Equations of state: w_Λ, $\dot{w}_\Lambda$
- ◇ Age, expansion rate: t_o, H_o
- ◇ Degree of inhomogeneity: A_S, n_S, A_T, n_T, σ_8
- ◇ Opacity: τ

Friedmann Equations.

We shall use two versions of Friedmann equations:

$$c^2 d(\rho a^3) = -p d a^3 \qquad \frac{\ddot{a}}{a} = -\frac{4\pi}{3} G(\rho + 3\frac{p}{c^2}) + \frac{\Lambda c^2}{3}$$

$$1 = \Omega_m + \Omega_k + \Omega_\gamma + \Omega_\Lambda \qquad \frac{\dot{a}^2}{a^2} + \frac{Kc^2}{a^2} = \frac{8\pi}{3} G(\rho_m + \rho_\gamma) + \frac{\Lambda c^2}{3}$$

Cosmographic Parameters.

• H: Hubble constant. Constant of proportionality between recession speed v and distance d in a expanding Universe.

$$v = H_o d \qquad H_o = 100h\ \mathrm{kms}^{-1}\mathrm{Mpc}^{-1} \qquad h = 0.73 \pm 0.04$$

subindex 'o' refers to quantities evaluated today. The Hubble constant CHANGES with time. From Friedmann's equations:

$$H(z) = \frac{\dot{a}}{a} = H_o E(z); \quad E(z) = (\Omega_\Lambda + \Omega_k(1+z)^2 + \Omega_m(1+z)^3 + \Omega_\gamma(1+z)^4)^{1/2};$$

• t_H: The inverse of H has units of time and gives the order of magnitude of the age of the Universe:

$$t_H = \frac{1}{H_o} = 9.78 \times 10^9 h^{-1} yr = 3.09 \times 10^{17} h^{-1} s$$

- Hubble Radius: $D_H = cH_o^{-1} = 3000h^{-1}Mpc$

- Critical Density: $\rho_c = 3H_o^2/8\pi G$. All densities are usually given in units of the critical density:

$$\Omega_\gamma = \frac{\rho_\gamma}{\rho_c}; \quad \Omega_b = \frac{\rho_b}{\rho_c}; \quad \Omega_\Lambda = \frac{\Lambda c^2}{3H_o^2} \qquad \Omega_{cdm} = \frac{\rho_{cdm}}{\rho_c};$$

$$\Omega_m = \Omega_m + \Omega_b \qquad \Omega_\Lambda + \Omega_m + \Omega_\gamma + \Omega_k = 1$$

- Redshift: Fractional Doppler shift due to the radial motion of the object.

$$1 + z = \frac{\nu_e}{\nu_o} = \frac{\lambda_o}{\lambda_e}$$

Angular Diameter and Luminosity Distances.

- D_A: Ratio of an object transverse physical size to its angular size

$$D_A = (1+z)^{-1} D_H \int_o^z \frac{dz'}{E(z')}$$

- D_L: Distance measured from the relation of bolometric flux S and bolometric luminosity L

$$D_L = \sqrt{\frac{L}{4\pi S}} \Longrightarrow D_L = (1+z)^2 D_A$$

- $k - correction$: If differential magnitudes on a frequency interval are used instead of bolometric magnitudes, then flux and luminosities of redshift objects need to be corrected because the redshifted object emits its flux at a wavelength that is different in which it is being observed.

Perfect Fluid Models.

♠ We shall describe the different energy-density components by means of a equation of state: $p = \omega\rho$. We distinguish:

Matter	$w = 0$	$\rho_m \sim a^{-3}$	$a(t) \sim t^{2/3}$
Radiation	$w = 0$	$\rho_\gamma \sim a^{-4}$	$a(t) \sim t^{1/2}$
Cosmological Constant	$w = -1$	$\rho_\Lambda \sim const.$	$a(t) \sim e^{Ht}$
Dark Energy	$-1 < w < -1/3$		
Phantom Energy	$w < -1$		

Problems of the Big-Bang Model.

Cosmological Parameters.

	$H_o/(\text{km/s Mpc}^{-1})$	$n_s(k = 0.002/\text{Mpc})$	$\Omega_b h^2$	Ω_Λ	Ω_m
WCDM+ALL	70.3 ± 1.6	0.946 ± 0.016	$0.02215^{+0.00070}_{-0.00072}$	0.745 ± 0.018	0.255 ± 0.018
WCDM+WMAP	100 (95% CL)	$0.954^{+0.018}_{-0.019}$	$0.02239^{+0.00075}_{-0.00079}$	$0.74^{+0.11}_{-0.12}$	$0.26^{+0.12}_{-0.11}$
	σ_8	t_o/Gyr	w	z_r	
WCDM+ALL	$0.712^{+0.044}_{-0.043}$	13.83 ± 0.15	$-0.926^{+0.054}_{-0.053}$	$10.3^{+2.7}_{-2.8}$	
WCDM+WMAP	$0.72^{+0.11}_{-0.12}$	$13.84^{+0.39}_{-0.36}$	$-1^{+0.45}_{-0.44}$	$10.9^{+2.6}_{-2.5}$	

TABLE of cosmological parameters for a Quintessence model with $w = const$ using all available astrophysical (ALL) data or only WMAP.

Problems with the Big-Bang Paradigm.

♠ Homogeneity: The Universe is on large scale homogeneous. The CMB is an almost perfect blackbody with no measured spectral distortions and with anisotropy at the level $\Delta T/T_o \sim 10^{-5}$.

♠ Large Scale Structure: At small scale the Universe is highly inhomogeneous, inhomogeneity that **is present on scales larger than the present particle horizon**.

♠ Flatness: The total matter of the Universe (Matter and Dark Energy) is close to 1.

♠ Coincidence: Matter and Dark energy densities, that evolve with time at very different rates, have very similar values today.

Dynamics of Friedmann models.

♠ Friedmann's equations for cosmological models described by **a single fluid with equation a of state $p = \omega\rho$** verifies the following scaling relations:

$$\rho \sim a^{-3(1+\omega)}; a \sim t^{2/3(1+\omega)} \quad \Rightarrow \quad \begin{cases} \omega = 0(MD) & a(t) \sim t^{2/3} \\ \omega = 1/3(RD) & a(t) \sim t^{1/2} \end{cases}$$

♠ The particle horizon represents, at each instant t the largest physical scale that is in casual contact with a given observer located at the (arbitrary) origin of coordinates. It is given by:

$$d_H(t) = a(t)\int_0^t \frac{dt'}{a(t')}; \quad if \quad a(t) \sim t^n \; with \; n < 1 \Rightarrow d_H(t) \sim t$$

The Horizon Problem.

♡ If $\omega = 1/3, 0$ the particle horizon grows faster than any physical length $l(t) = a(t)l_o$. The scale of the horizon today was larger than the horizon at any prior epoch. If $d_H(t_o) = l(t_o)$ then

$$d_H(t) = d_H(t_o)\frac{t}{t_o} = l(t_o)\left(\frac{a(t)}{a(t_o)}\right)^{1/n} = l(t)\left(\frac{a(t)}{a(t_o)}\right)^{1/n-1}$$

and if $n < 1$ then $d_H(t) < l(t)$.

♡ The horizon problem is a problem about initial conditions. How came about that the Universe is Homogeneous today if the observable Universe was outside the horizon in any MD or RD period.

The Flatness Problem.

Friedmann equation states:

$$\Omega(t) - 1 = \frac{K}{(aH)^2} = (\Omega_o - 1)\left(\frac{a_o H_o}{a(t)H(t)}\right)^2$$

If $a(t) \sim T_{CMB}^{-1}$, $H(t) \sim t^{-1}$, at the Planck epoch:

$$\Omega(t_{Pl}) - 1 = (\Omega_o - 1)\left(\frac{T_o t_o}{T_{Pl} t_{Pl}}\right) = (\Omega_o - 1) \times 10^{-58}$$

This problem was solved introducing a period of acelerated expansion (termed inflation) where the dynamics was dominated by scalar fields.

The Coincidence Problem.

WMAP Measured Cosmological Paramenters:

$$\Omega_b = 0.046, \qquad \Omega_c = 0.191, \qquad \Omega_\Lambda = 0.763, \qquad \Omega_\gamma = 5 \times 10^{-5}$$

Coincide Problem: two magnitudes (ρ_c and $\rho_\Lambda = \Lambda/8\pi G$) that scale very differently with time, have similar values today.

$$\frac{\Lambda}{\rho_c(t_{Pl})} = \frac{\Lambda}{\rho_c(t_o)} \left(\frac{m_{Pl}}{T_o}\right)^{-3} \qquad \Rightarrow \qquad \frac{\Lambda}{\rho_c(t_{Pl})} \sim 10^{-95}$$

$(m_{Pl} \sim 10^{19} GeV, T_o = 2.5 \times 10^{-4} eV)$

Cosmological Constant vs. Dark Energy.

The Cosmological Constant.

The "modified" Einstein equations are:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

The cosmological constant can be introduced as a perfect fluid with equation of state:

$$\rho = -p = \frac{\Lambda}{8\pi G}$$

Scalar fields in Cosmology.

High Energy Physics provides candidates for DE in terms of scalar fields.

$$\mathcal{L} = \mathcal{L}(\phi; \phi_{,\nu}; \mathrm{x}^{\nu}) = \frac{1}{2}\dot{\phi}^2 - \mathrm{V}(\phi)$$

Lagrangian densities allow to define the energy-momentum tensor . . .

$$T_{\mu\nu} = \frac{\partial \mathcal{L}}{\partial \phi_{,\nu}}\phi_{,\nu} - \mathcal{L}\eta_{\mu\nu} \qquad \Longrightarrow \qquad T_{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}$$

. . . and the corresponding equation of state is

$$\left.\begin{array}{l} \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ p = \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{array}\right\} \quad \text{if and only if} \quad V(\phi) \gg \dot{\phi}^2 \quad \Longrightarrow \quad p = -\rho$$

$\longrightarrow$ equivalent to a Cosmological Constant

Today $\Omega_\Lambda \sim 1 \Rightarrow \Lambda \sim 3H_o^2$. The corresponding energy density is: $\rho_\Lambda = 10^{-47} GeV^4$. The vacuum energy density at Planck time from all matter fields is $\rho_{vac} = 10^{74} GeV^4$,

121 orders of magnitude larger!!!

Cosmological constant vs. Dark Energy.

The equation of state of the dominant energy component at present is not known:

$$p = w\rho$$

Observational results: $w = -0.81 \pm 0.22$ (WMAP alone) $w = -0.915 \pm 0.051$ (including several data sets).

Different nomenclature:

1. Cosmological constant: $w = -1$.
2. Dark Energy: $-1 < w < -1/3$.
3. Phantom Energy: $w < -1$.

Scalar fields / Dark Energy Models

Given a scalar field lagrangian we have a model of Dark Energy. If $X = 1/2g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$, then

1. Quintessence: $\mathcal{L} = X(\phi) - V(\phi)$
2. K-essence: $\mathcal{L} = f(\phi)(X + X^2)$
3. Tachyon: $\mathcal{L} = V(\phi)\sqrt{-det(g_{ab} + \partial_a\phi\partial_b\phi)}$
4. Born-Infeld: $\mathcal{L} = V(\phi)\sqrt{1 - 2M^{-4}X}$
5. Phantom: $\mathcal{L} = -X(\phi) - V(\phi)$
6. Chaplygin gas: $p = -A/\rho$

Problems.

• Let us assume that the matter content of the Universe has two components: ordinary matter and a cosmological constant with equation of state parameters $\omega = 0$ and $\omega = -1$, respectively. Assuming the Universe is flat ($\Omega_m + \Omega_\Lambda = 1$), calculate the age of the Universe in closed form, in terms of $\Omega_m(t_o)$ and H_o.

• Look up in the literature what was, in the 90's, the Globular Cluster age problem.