The Inflation Solution.

Accelerated Expansion.

♡ As long as 'gravity' dominates the dynamics, from Friedmann equation we have that the expansion is decelerated:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

If $\rho + 3p = 0$ then $a(t) \sim t$, scale length grow at the same rate as the horizon.

♡ If the energy density content of the Universe is well described by a perfect fluid with equation of state: $p = \omega\rho$ then, for accelarated expansion:

$$\rho + 3p < 0 \qquad \Longrightarrow \qquad \omega < -\frac{1}{3}$$

In the standard Big-Bang model $\omega = 1/3, 0$ and there is a particle horizon and a horizon problem.

Scalar Fields (again!!).

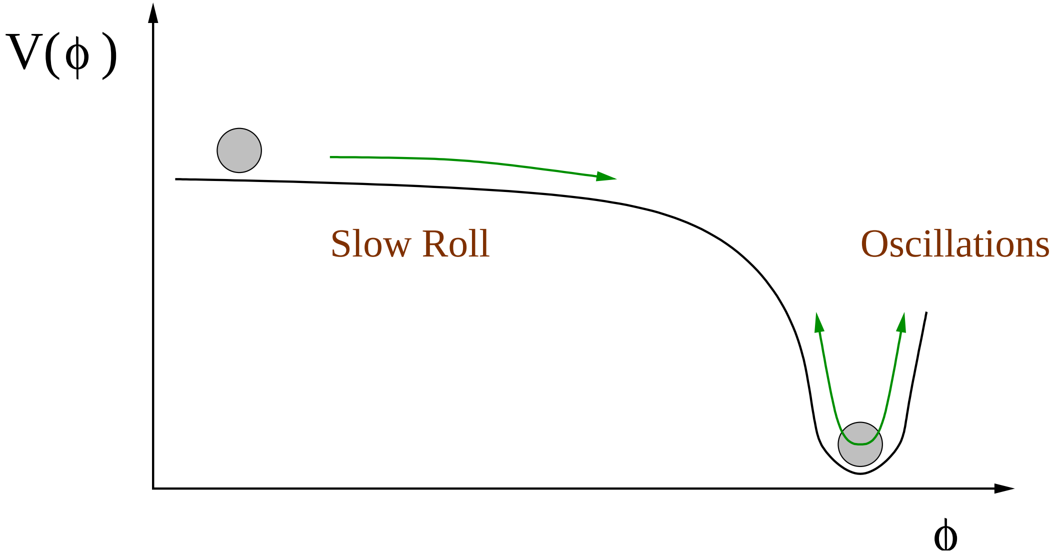
◇ For a scalar field of Klein-Gordon type we have:

$$\left.\begin{array}{l}\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ p = \frac{1}{2}\dot{\phi}^2 - V(\phi)\end{array}\right\} \quad \text{if} \quad V(\phi) \gg \dot{\phi}^2 \qquad \Longrightarrow \qquad p = -\rho$$

◇ Further, if $V(\phi) \approx const.$ then

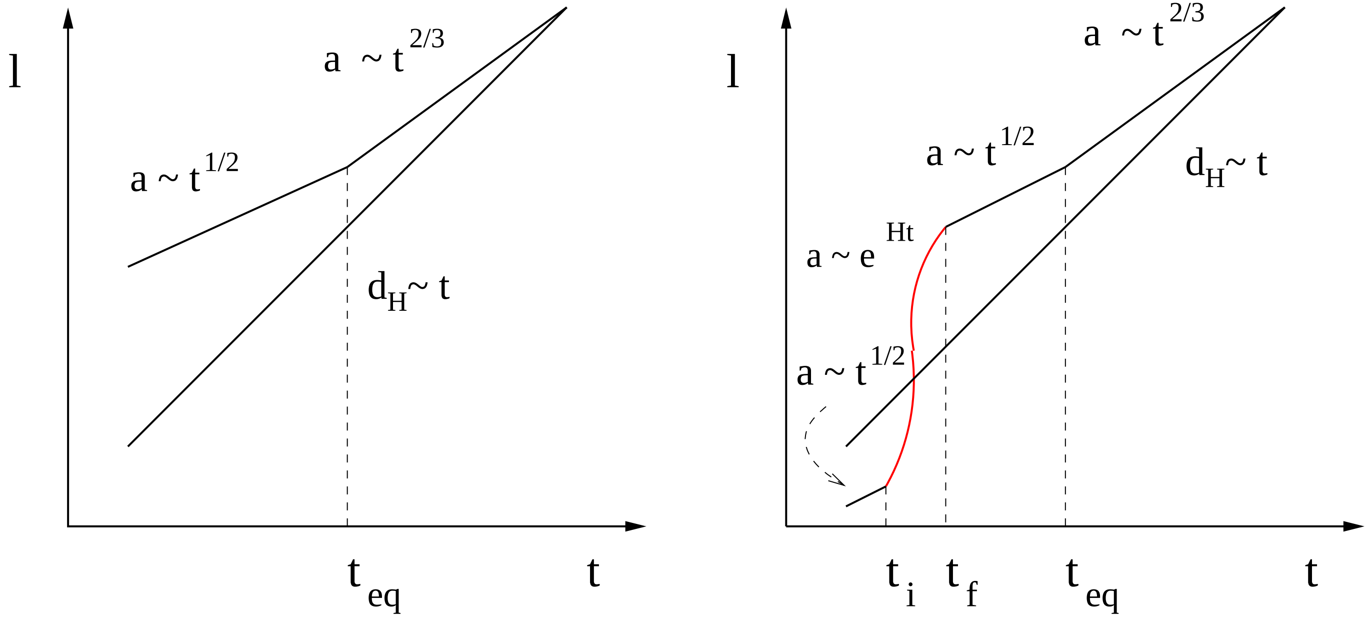
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \quad \Rightarrow \quad \frac{\dot{a}}{a} \approx \sqrt{\frac{8\pi G}{3}V(\phi)} = const = H \quad \Rightarrow \quad a(t) = e^{H(t-t_i)}$$

The Inflation Paradigm.



During the period when the scalar field dominates the dynamics, there is a 'slow-roll' phase when $\dot{\phi}^2 \ll V(\phi)$, and $p = -\rho$, followed by a period of 'reheating' when the scalar field decays and the Universe becomes once more radiation dominated.

Evolution of the Scale Factor.



Growth rate of a physical distance of two comoving points and the horizon scale, as a function of time. Left: Standard Big-Bang Model. Right: Big-Bang model with an inflation epoch. Inflation starts at t_i and ends at t_f.

The Inflation Solution of the Horizon and Flatness Problems.

♣ A period of exponential expansion solves both the horizon and flatness problem. Let us assume that inflation starts at t_i and ends at t_f. Let impose that inflation last long enough to solve the horizon problem, then two scale lengths l_i and l_o will coincide with the horizon at the start of inflation and horizon today. At those time the the horizon is $d_H(t_i) = cH_i^{-1} = ct_i$ and $d_H(t_o) = cH_o^{-1} = ct_o$. Then:

$$d_H(t_i) = l_i = \frac{a_i}{a_o} l_o = \frac{a_i}{a_o} d_H(t_o) \quad \Rightarrow \quad \boxed{H_i a_i = H_o a_o}$$

♣ The previous condition solves also the Flatness problem. Friedmann equation can be rewritten as:

$$\frac{k}{a^2 H^2} = \Omega - 1 \quad \Rightarrow \quad (\Omega_i - 1)^{1/2} = (\Omega_o - 1)^{1/2} \frac{H_o a_o}{H_i a_i}$$

Length of the Inflationary Period.

♠ For inflation to produce enough expansion to explain the Horizon and Flatness problems, inflation must verify:

$$H_i a_i = H_o a_o \quad \Rightarrow \quad t_i = t_o \frac{a_i}{a_{eq}} \frac{a_{eq}}{a_f} \frac{a_f}{a_o} \quad \Rightarrow \quad \frac{t_o}{t_i} = \left(\frac{t_o}{t_{eq}}\right)^{2/3} \left(\frac{t_{eq}}{t_f}\right)^{1/2} e^{H_i(t_f - t_i)}$$

Therefore, if $t_i \sim t_{Pl} \sim 10^{-43} s$, the number of e-foldings $N = H_i(t_f - t_i)$ must be:

$$N = \ln \left[\frac{t_o}{t_i} \left(\frac{t_{eq}}{t_o}\right)^{2/3} \left(\frac{t_f}{t_{eq}}\right)^{1/2} \right] \approx \ln \left[\frac{t_{eq}^{1/6} t_o^{1/3}}{t_i^{1/2}} N^{1/2} \right] \approx 60$$

♠ The number of e-foldings is related to the flatness of the inflaton potential in the 'slow-roll' regime. This constrain is so difficult to realize that there are not known candidates for the inflaton field.

Problems.

• The solution to the horizon problem provided by inflation is temporary. After a while, the Univers will become inhomogeneous. Let us assume that inflation starts at t_i and ends at t_f. show that the observed universe homogeneous only for a time $t < t_{crit}$ where

$$t_{crit} \simeq \frac{t_i^2}{t_f} \exp[2H(t_f - t_i)]$$