Problem 1.

• Let us assume that the matter content of the Universe has two components: ordinary matter and a cosmological constant with equation of state parameters $\omega = 0$ and $\omega = -1$, respectively. Assuming the Universe is flat ($\Omega_m + \Omega_\Lambda = 1$), calculate the age of the Universe in closed form, in terms of $\Omega_m(t_o)$ and H_o.

• Look up in the literature what was, in the 90's, the Globular Cluster age problem.

Problem 2.

• The solution to the horizon problem provided by inflation is temporary. After a while, the Univers will become inhomogeneous. Let us assume that inflation starts at t_i and ends at t_f. show that the observed universe homogeneous only for a time $t < t_{crit}$ where

$$t_{crit} \simeq \frac{t_i^2}{t_f} \exp[2H(t_f - t_i)]$$

Problem 3.

- The variance of the matter density field is

$$\sigma_M^2(R) = \langle \left(\frac{\delta M}{M}\right)^2_R \rangle = \int_o^\infty \frac{dk}{k} W_k^2 \frac{k^3}{2\pi^2} P(k)$$

For a power law spectrum $P(k) = Ak^n$, the integral can be evaluated analytically. Show that $\sigma_M(R) \propto R^{-(n+3)/2} \propto M^{-(n+3)/6}$.

- WMAP normalization of the power spectrum requires $\sigma_M(R = 8h^{-1}Mpc) = \sigma_8 = 0.8$. The 'Great Attractor' is a region of $\sim 50h^{-1}Mpc$ with an overdensity $(\delta M/M) \sim 0.7$. What is the probability of finding a 'Great Attractor' region if the power spectrum is $n = 0, -1, -2$?. How many 'Great Attractor' regions will we find within the current horizon?.

Problem 4.

♠ (a) Consider an electron whichis moving with a velocity v through a radiation bath of temperature T. Show that the electron will feel a 'drag' force

$$F = -\frac{4\pi^2}{15}\sigma_T T^4 v$$

where σ_T is Thomson cross section. (b) Assume now that a bath of electrons of temperature T_e crosses the bath of photons. Show that the net rate of transfer of energy from radiation to matter per electron is

$$\frac{dQ}{dt} = \frac{4\pi^2}{15}\sigma_T \left(\frac{T^4}{m}\right)(T - T_e)$$

(c) It the matter is fully ionized, then the kinetic energy per electron is $3T_e$. Therefore, using the previous expression, show that the change in temperature is

$$\dot{T}_e = \frac{4\pi^2}{45}\sigma_T \left(\frac{T^4}{m}\right)(T - T_e)$$

How would this equation be modified in a expanding Universe, with only a fraction of the matter ionized.