The Harrison-Zel'dovich Power Spectrum.

Notation and earlier results.

t_{in}: moment when a perturbation comes into the horizon.

t_{eq}: moment of matter radiation equality; $\rho_\gamma(t_{eq}) = \rho_m(t_{eq})$.

t_o: present time.

k, λ: Comoving wavenumber and wavelength; $k = 2\pi/\lambda$.

$\Delta(k,t)$: amplitude of a perturbation of given wavenumber at time t:

$$\Delta^2(k,t) = D_+(t)\frac{k^3}{2\pi^3}P(k)$$

$D_+(t)$: growth factor.

$$D_+(t) = \begin{cases} D_+(t_{in})(t/t_{in})^{2/3} & if \quad t_{in} > t_{eq} \\ D_+(t_{in}) = const. & if \quad t_{in} \leq t_{eq} \end{cases}$$

♣ We define the moment a perturbation comes within the horizon when its physical length coincides with the size of the horizon:

$$\lambda(t_{in})a(t_{in}) = d_H(t_{in})$$

Since $d_H(t) \sim t$ and during MATTER DOMINATION $a(t) \sim t^{2/3}$, then $\lambda(t_{in}) \sim t_{in}^{1/3}$. Therefore:

$$k_{in}^3 t_{in} = const = k_{eq}^3 t_{eq}$$

♣ Harrison (1970) & Zeldovich (1972) prescription.

ALL PERTURBATIONS THAT COME WITHIN THE HORIZON HAVE THE SAME AMPLITUDE

$$D_+(t_{in})\Delta(k_{in}) = const = D_+(t_{eq})\Delta(k_{eq}) \qquad \forall t_{in}$$

This prescription gives the amplitude of the matter power spectrum at different moments in time.

Matter power spectrum at any given time.

♠ Let us assume that that the perturbation comes into the horizon during RD: $t_{in} < t_{eq}$; taking into account

$$D_+(t_{in}) = D_+(t_{eq}) \qquad \text{and} \qquad D_+(t_{in})\Delta(k_{in}) = D_+(t_{eq})\Delta(k_{eq})$$

then

$$D_+(t_{eq})\Delta(k_{in}) = D_+(t_{eq})\Delta(k_{eq}) \qquad \Longrightarrow \qquad D_+(t_o)\Delta(k_{in}) = D_+(t_o)\Delta(k_{eq})$$

If a perturbation comes into the horizon before Matter–Radiation equality, its amplitude today is related to the amplitude at MR eq as:

$$P(k_{in}) = P(k_{eq}) \left(\frac{k_{in}}{k_{eq}}\right)^{-3}$$

♠ Let us assume that a perturbation comes into the horizon at a time $t_{in} > t_{eq}$; i.e, during MD.

$$D_+(t_o) = D_+(t_{in}) \left(\frac{t_o}{t_{in}}\right)^{2/3} = D_+(t_{in}) \left(\frac{t_o}{t_{eq}}\right)^{2/3} \left(\frac{t_{eq}}{t_{in}}\right)^{2/3}$$

therefore

$$\begin{aligned} D_+(t_o)\Delta(k_{in}) &= D_+(t_{in})\Delta(k_{in}) \left(\frac{t_o}{t_{eq}}\right)^{2/3} \left(\frac{t_{eq}}{t_{in}}\right)^{2/3} = D_+(t_{eq})\Delta(k_{eq}) \left(\frac{t_o}{t_{eq}}\right)^{2/3} \left(\frac{t_{eq}}{t_{in}}\right)^{2/3} \\ &= D_+(t_o)\Delta(k_{eq}) \left(\frac{t_{eq}}{t_{in}}\right)^{2/3} = D_+(t_o)\Delta(k_{eq}) \left(\frac{k_{eq}}{k_{in}}\right)^{2} \end{aligned}$$

and finally

$$D_+(t_o)\Delta(k_{in})k_{in}^2 = const = D_+(t_o)\Delta(k_{eq})k_{eq}^2$$

therefore, in this case if a perturbation comes into the horizon AFTER Matter–Radiation equality, its amplitude today is related to the amplitude at MR eq as:

$$P(k_{in}) = P(k_{eq}) \left(\frac{k_{in}}{k_{eq}}\right)^{1}$$

Redshift Surveys. Observations of the Matter Power Spectrum.

Redshift Surveys.

♠ Matter Power Spectrum can be obtained from Galaxy Redshift Surveys. Recent Surveys: 2dFRGS and SDSS.

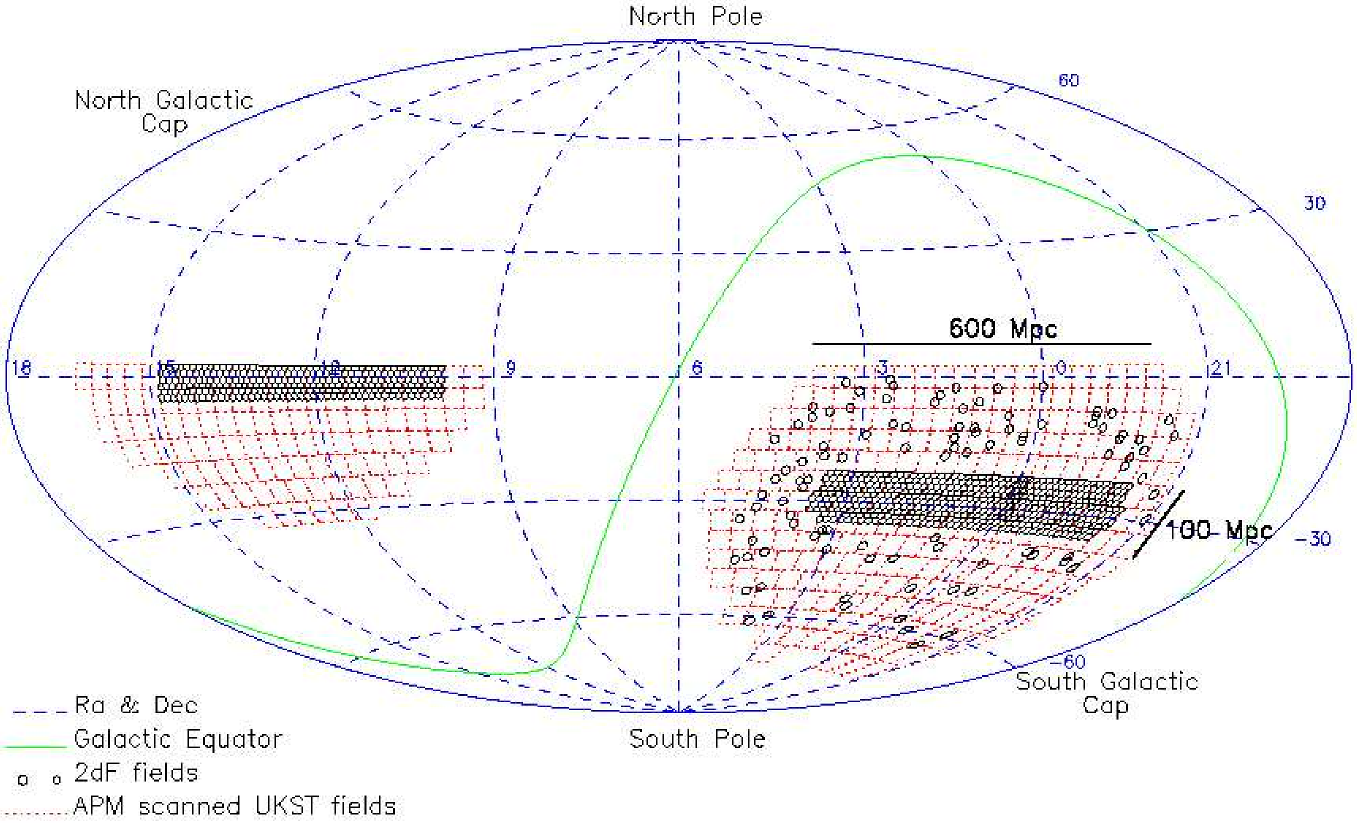
♠ Redshift surveys of galaxies of different types provide information about the matter distribution and can be used to derive the matter power spectrum.

♠ There are very different type of surveys: 1D (pencil-beam), 2D (slices), 3D; targetted and blind, flux or volume limited.

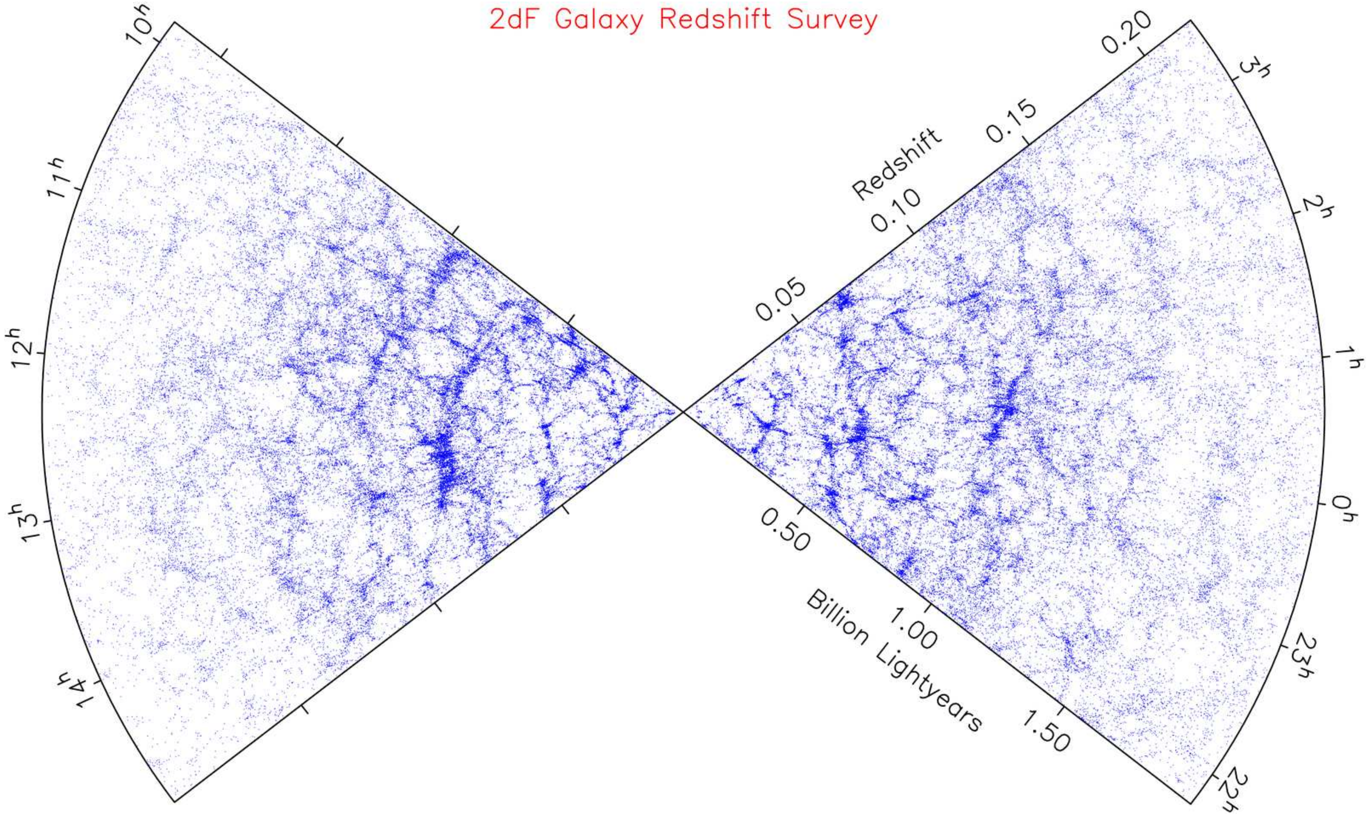
♠ In targetted surveys, only a specific class of objets is searched. In blind surveys, the whole sky is sampled and objects are later detected.

♠ In flux limited samples the number density of objects drops as a function of their distance from the Earth; the drop is quantified through the selection function.

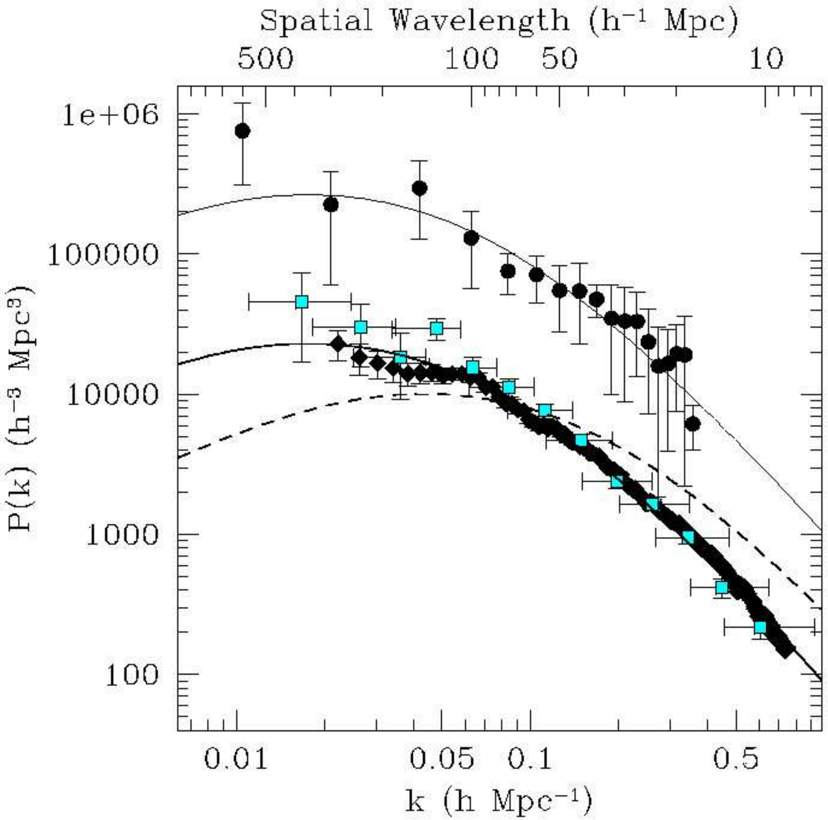
2dF Survey Map.



2dF Cone.


2dF Galaxy Redshift Survey
10h
11h
12h
13h
14h
3h
2h
1h
0h
23h
22h
Redshift
0.05
0.10
0.15
0.20
Billion Lightyears
0.50
1.00
1.50

2dF Matter Power Spectrum.



Galaxy Luminosity Function.

- Luminosity function: density density of galaxies of luminosity L per unit of luminosity:

$$\Phi(L) = \frac{d^4 N(L)}{dV\,dL} \qquad \Longrightarrow \qquad \bar{n} = \int \Phi(L) dL$$

- In practice, redshift surveys only detect objects above some f_{min} flux. The luminosity function is poorly constrained in the faint end, so the integral is cut off at some lower limit $L_S = 4\pi r_S^2 f_{min}$, for some small $r_s \sim 50h^{-1}$Mpc.

- At any distance $r > R_S$ we can only observe galaxies with luminosities greater than:

$$L_{min} = 4\pi r^2 f_{min}$$

Survey Selection Function.

• In flux limited redshift surveys the number density of sample objects is a decreasing function of reshift. This decrement is quantified in terms of the **selection function** $\phi(r)$ defined as the fraction of galaxies at distance r that meed the sample selection criteria.

• In order to use galaxies in a sample as fair tracers of the general galaxy distribution, we must correct the fact that galaxies in the sample represent a smaller fraction of the parent population at larger distances.

• If f_{min} is the limiting flux of the survey, the survey selection function is the ratio of the observed galaxies to the total number of galaxies at distnace r:

$$\phi(r) = \frac{\int_{4\pi r^2 f_{min}}^{\infty} dL \Phi(L)}{\int_{L_S}^{\infty} dL \Phi(L)}$$

The Schechter Luminosity Function.

References.

- Kaiser (1984) ApJ 284, L9
- BBKS (1986) ApJ 304, 15