Recombination.

CMB and Big Bang Nucleosynthesis.

♠ To explain the observed He fraction is necessary that BBN be delayed and started with the production of D at $T \approx 0.1$MeV. That temperature corresponds to a time scale: $t = [1\mathrm{MeV}/T]^2 \approx 100$s.

♠ The cross section of strong interaction at that temperature is:

$$\sigma v(p + n \rightarrow D + \gamma) \simeq 5 \times 10^{-20}\mathrm{cm}^3/\mathrm{s}$$

♠ If the reaction rate is $\Gamma = t_{mfp}^{-1} = n_B \sigma v \sim 1$ then at that temperature: $n_B = 2 \times 10^{17}\mathrm{cm}^{-3}$

♠ The abundance of baryons today is $n(t_o) \sim a^{-3} \sim T_o^3$. Therefore:

$$n_B(t_o) T_o^{-3} = n_B(t_{BBN}) T_{BBN}^{-3} \qquad \Longrightarrow \qquad T_o = \left[\frac{n_B(t_{BBN})}{n_B(t_o)}\right]^{1/3} T_{BBN} \simeq 10\mathrm{K}$$

He synthesis in the early Universe requires the existence of a CMB of temperature $T \simeq 10\mathrm{K}$

Abundances of Different Particle Species in Thermal Equilibrium.

◇ Number density of relativistic particles

$$n = \frac{\zeta(3)}{\pi^2} g T^3 \qquad (boson)$$

$$n = \frac{3}{4}\frac{\zeta(3)}{\pi^2} g T^3 \qquad (fermion)$$

◇ Number density of non-relativistic particles, valid for both bosons and fermions:

$$n = g\left(\frac{mT}{2\pi}\right)^{3/2} \exp\left[-\frac{1}{T}(m-\mu)\right]$$

Recombination.

◇ At $T \leq 13.6$eV neutral atoms may form. At that temperature, (p,e,H) are non-relativitic and their thermal abundances are given by:

$$n_i = g_i \left(\frac{m_i T}{2\pi} \right) \exp \left[\frac{\mu_i - m_i}{kT} \right]$$

where T is the equilibrium temperature common to all particles in the reaction and μ_i the chemical potential.

◇ The recombination reaction: $p + e \rightleftharpoons H + \gamma$ fixes $\mu_p + mu_e = \mu_H$ and yields:

$$n_H = \frac{g_H}{g_e g_p} n_p n_e \left(\frac{m_e T}{2\pi} \right)^{-3/4} \exp \left[\frac{B}{kT} \right]$$

where $B = m_p + m_e - m_H = 13.6$eV and $m_H \approx m_p$ except on the exponential.

If $x_e = n_e/n_B$ denotes the fraction of free electrons, then

$$\left.\begin{array}{l} n_H = n_B - n_p = \eta n_\gamma - n_e \\ g_\gamma = g_p = g_e = g_H/2 = 2 \\ n_\gamma = \frac{\zeta(3)}{\pi^2} g_\gamma T^3 \end{array}\right\} \Longrightarrow \frac{1-x_e}{x_e^2} = \sqrt{\frac{32}{\pi}}\zeta(3)\eta\left(\frac{T}{me}\right)^{3/2}\exp\left[\frac{B}{kT}\right]$$

Saha's Equation

We call RECOMBINATION the moment when 90% of the electrons are bound into neutral atoms.
If $x_e = 0.1$ then:

$$\eta = 2.68 \times 10^{-8}(\Omega_B h^2) \quad \Longrightarrow \quad 3.13 \times 10^{-18}(\Omega_B h^2)\left(\frac{1eV}{T}\right)^{3/2} = \exp\left[13.6\frac{1eV}{T}\right]$$

This equation must be solved by iteration.

A zero order solution is:

$$\frac{1eV}{T} = 3.084 - 0.0735\ln(\Omega_B h^2) \quad \Longrightarrow \quad \begin{cases} T \sim 0.3eV \ll 13.6eV \\ (1_z)_{rec} \approx 1367[1 - 0.024\ln(\Omega_B h^2)]^{-1} \end{cases}$$

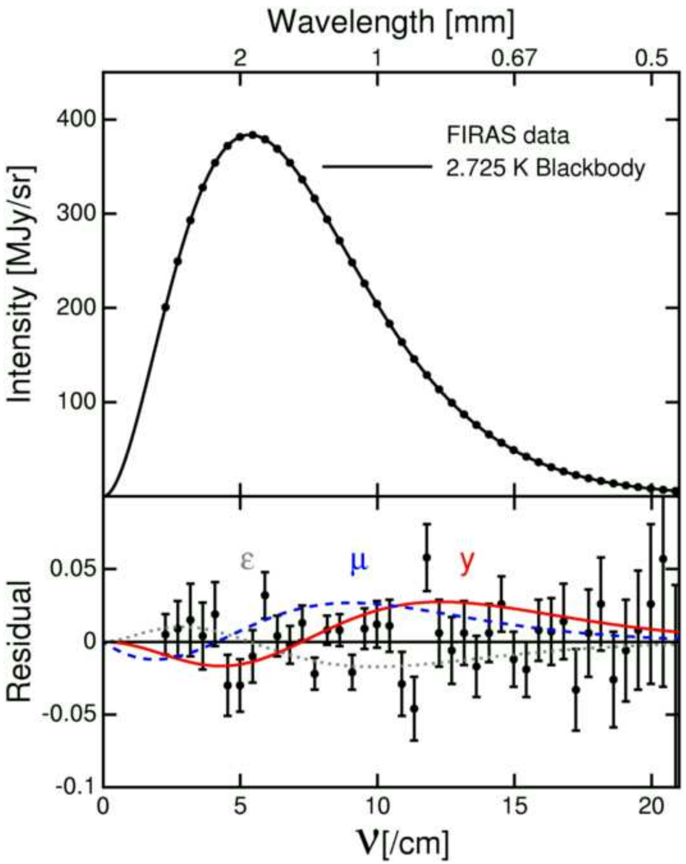
The very small baryon to photon ratio η delays the formation of neutral atoms from the expected temperature of 13.6eV to $T \sim 0.3$eV.

Decoupling.

Mean free path between photon electron collisions for different processes:

1. Thomson scattering; NO energy exchange and dominant at low redshifts:
$t_{Th} = (c n_e \sigma_T)^{-1} = 6.14 \times 10^7 s (T/1eV)^{-3} (x_e \Omega_B h^2)^{-1}$
2. Compton scattering; Photon-electron energy exchange.
$t_{Comp} = (n_e \sigma_T)^{-1} (mc/T) = 3.0 \times 10^{13} (T/1eV)^{-4} (x_e \Omega_B h^2)^{-1}$
3. Free-Free absorption:
$t_{ff} = 1.1 \times 10^{11} s (\Omega_B h^2 x_e)^{-3/2} (T/1eV)^{-11/4}$
4. Double Compton scattering $(e + \gamma \rightarrow e + 2\gamma)$:
$t_{2c} = 10^{20} s (T/1eV)^{-5} (\Omega_B h^2)^1$

Spectral Distortions.



Problems.

♠ (a) Consider an electron whichis moving with a velocity v through a radiation bath of temperature T. Show that the electron will feel a 'drag' force

$$F = -\frac{4\pi^2}{15}\sigma_T T^4 v$$

where σ_T is Thomson cross section. (b) Assume now that a bath of electrons of temperature T_e crosses the bath of photons. Show that the net rate of transfer of energy from radiation to matter per electron is

$$\frac{dQ}{dt} = \frac{4\pi^2}{15}\sigma_T\left(\frac{T^4}{m}\right)(T - T_e)$$

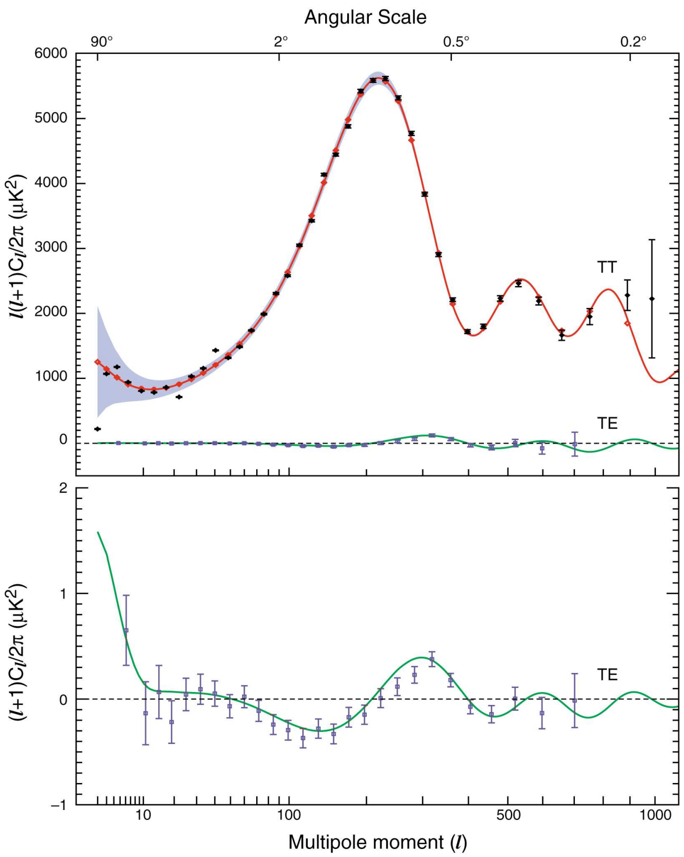
(c) It the matter is fully ionized, then the kinetic energy per electron is $3T_e$. Therefore, using the previous expression, show that the change in temperature is

$$\dot{T}_e = \frac{4\pi^2}{45}\sigma_T \left(\frac{T^4}{m}\right)(T - T_e)$$

How would this equation be modified in a expanding Universe, with only a fraction of the matter ionized.

THE RADIATION POWER SPECTRUM.

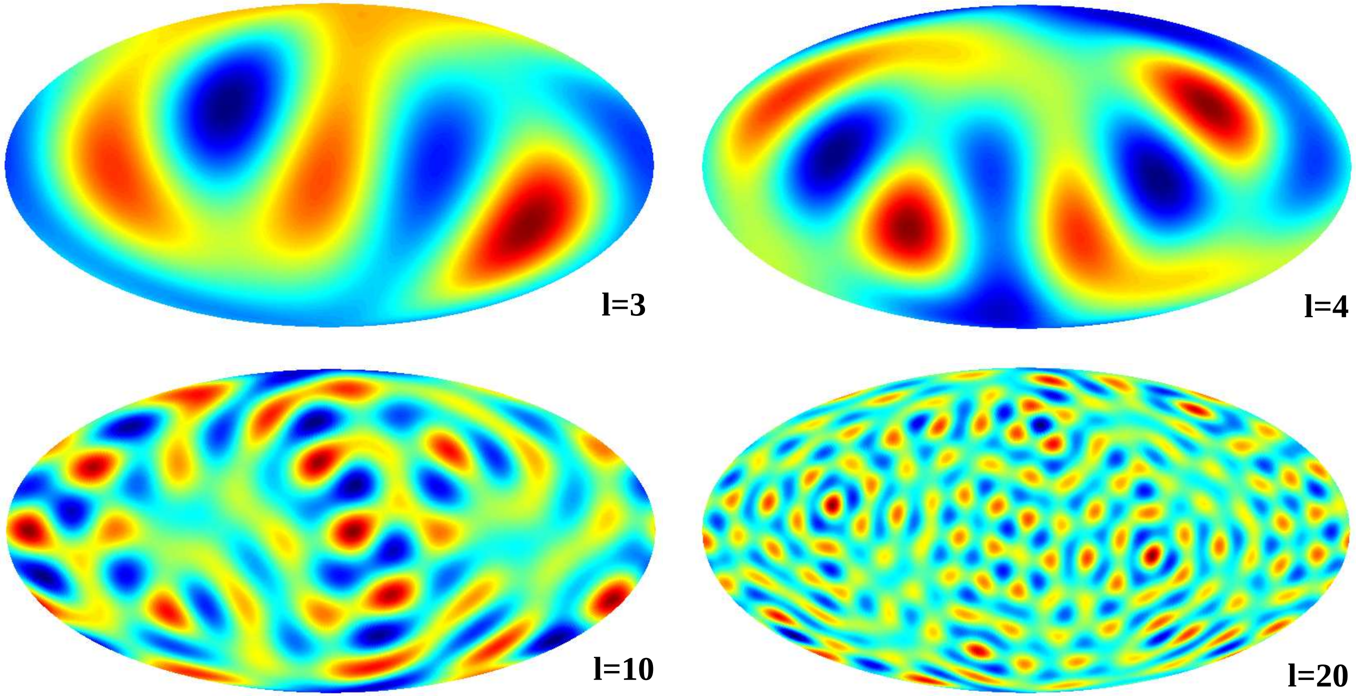
TT-TE Power Spectrum.



The radiation power spectrum has very distinctive features:

1. Plateau at low multipoles
2. Peaks of different height
3. Trough of similar height
4. Damping at high multipoles

Multipoles.



Each Multipole is associated with a distinctive angular scales.

Angles & Multipoles.

♣ Any spherical harmonic Y_{lm} has $2l+1$ zeros in the azimuthal direction and l zeros in the polar direction. However, the angular scales in the azimuthal direction vary with latitude. An approximate angular scale can be obtained as:

$$\Omega = \frac{4\pi}{l(2l+1)} \approx \frac{2\pi}{l^2} \quad \Rightarrow \quad \alpha_l \sim \sqrt{\Omega} = \frac{\sqrt{2\pi}}{l} = \frac{144^o}{l}$$

Physics of the Baryon-Photon Plasma.

- Ma & Bertschinger (1996) ApJ; Hu & Dodelson (2002) ARAA.

♣ Prior to recombination baryons and photons are strongly coupled. The effect of baryons is to exchange energy-momentum between photons at different frequencies. We can neglect the effect of gravity.

♣ Boltzmann equation shows that the monopole and dipole moments of the temperature field in the previous approximation is:

$$\dot{\Theta}_0 = -\frac{4}{3}\Theta_1 \qquad \dot{\Theta}_1 = -\frac{k^2}{3}\Theta_0$$

where derivatives are with respect to conformal time.

Accoustic Oscillations

♣ Since the sound speed is $c_s = \sqrt{\dot{p}/\dot{\rho}} = 1/\sqrt{3}$, both equations can be coupled to give:

$$\ddot{\Theta}_0 + c_s^2 k^2 \Theta_0 = 0$$

Physically, these temperature oscillations represent the heating and cooling of a fluid that is compressed and rarefied by an standing accoustic wave. This behaviour continues until recombination.

♣ The temperature distribution at recombination will be:

$$\Theta(k, \tau_{rec}) = A \cos(k c_s \tau_{rec})$$

In here: $c_s \tau_{rec}$ is the distance sound can travel by the time τ_{rec}, called sound horizon.

Peak Structure.

♣ The radiation power spectrum is the variance of $\Theta(k, \tau_{rec})$; therefore, modes that are caught at maxima or minima of their oscillation at recombination correspond to peaks in the power spectrum. This is the origin of the 'peaks' detected in the observations.

♣ How does this spectrum of inhomogeneities at recomgination appear to us today?. A wavelength λ appears as an angular anisotropy of scale $\theta \approx \lambda / d_A$, with d_A the angular diameter distance. Therefore, peaks should appear at

$$l_n \sim n \frac{\pi d_{A,rec}}{c_s \tau_{rec}}$$

Therefore, the position of the accoustic peaks can be used to determine the geometry of the Universe.

♣ At small scales, the coupling of baryons and photons is not so perfect and photons leak out and erase density perturbations (SILK DAMPING).

Parameter Sensitivity.

♡ If we denote by $l_a = \pi d_{A,rec}/c_s\tau_{rec}$ the multipole corresponding to the angular extend of the sound horizon, by $l_{eq} = k_{eq} d_{A,rec}$ the particle horizon at matter-radiation equality and l_d the Silk damping scale, then for a large class of cosmological models, cosmological parameters are approximately given by:

$$\begin{aligned}
\frac{\Delta l_a}{l_a} &\approx -0.24\frac{\Delta\Omega_m h^2}{\Omega_m h^2} + 0.07\frac{\Delta\Omega_b h^2}{\Omega_b h^2} - 0.17\frac{\Delta\Omega_\Lambda}{\Omega_\Lambda} - 1.1\frac{\Delta\Omega_{tot}}{\Omega_{tot}} \\
\frac{\Delta l_{eq}}{l_{eq}} &\approx 0.5\frac{\Delta\Omega_m h^2}{\Omega_m h^2} + -0.17\frac{\Delta\Omega_\Lambda}{\Omega_\Lambda} - 1.1\frac{\Delta\Omega_{tot}}{\Omega_{tot}} \\
\frac{\Delta l_d}{l_d} &\approx -0.21\frac{\Delta\Omega_m h^2}{\Omega_m h^2} + 0.20\frac{\Delta\Omega_b h^2}{\Omega_b h^2} - 0.17\frac{\Delta\Omega_\Lambda}{\Omega_\Lambda} - 1.1\frac{\Delta\Omega_{tot}}{\Omega_{tot}}
\end{aligned}$$

CMB TEMPERATURE ANISOTROPIES.

Sources of CMB Anisotropy.

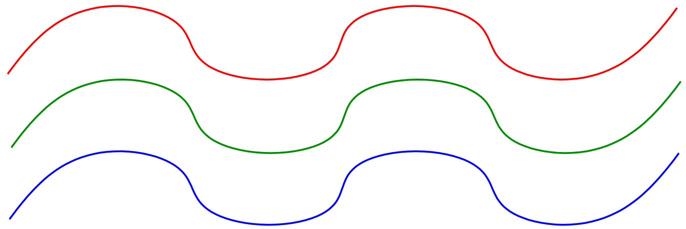
$$\frac{\Delta T(\vec{x_o},\vec{n})}{T_o} = \underbrace{\frac{1}{3}(\Phi(\vec{x_e}) - \Phi(\vec{x_o}))}_{Sachs-Wolfe} + \underbrace{(\vec{v}(\vec{x_e}) - \vec{v}(\vec{x_o}))\vec{n}}_{Doppler} - \underbrace{\frac{2}{c^2}\int_e^o dx \frac{\partial \Phi(x\vec{n})}{\partial x}}_{Integrated\ Sachs-Wolfe} + \underbrace{\frac{\Delta T(\vec{x_e})}{T_o}}_{Intrinsic}$$

♠ Intrinsic Fluctuations.

$$\rho_\gamma \sim a^{-4} \sim T^4 \qquad \Longrightarrow \qquad \delta_\gamma = \frac{\delta\rho_\gamma}{\rho_\gamma} = 4\frac{\delta T}{T_o} \qquad \Longrightarrow \qquad \frac{\delta T}{T_o} = \frac{1}{4}\delta_\gamma$$

If the density of the radiation field varies across the sky, it will generate temperature anisotropies. The relation between the different energy densities defines the **Fluctuation Type**.

Adiabatic Fluctuations.



The relative number density of particles is the same at every point for all particle species.

$$\delta = \frac{\delta n_B}{n_B} = \frac{\delta n_X}{n_X} = \frac{\delta n_\gamma}{n_\gamma}$$

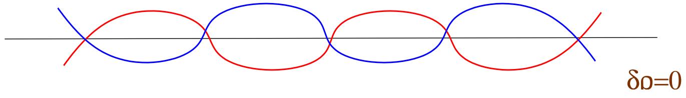
Since $a^{-1} \sim T$, $n_B \sim a^{-3}$ and $n_\gamma \sim a^{-4}$ then:

$$\delta_\gamma = \frac{4}{3}\delta_B; \qquad \frac{\delta T}{T_o} = \frac{1}{4}\delta_\gamma = \frac{1}{3}\delta_B$$

This fluctuations are termed **Adiabatic** because the baryon/photon ratio does not change anywhere, i.e., there is no heat flux:

$$\delta\left(\frac{n_B}{n_\gamma}\right) = \frac{n_B}{n_\gamma}\left[\frac{\delta n_B}{n_B} - \frac{\delta n_\gamma}{n_\gamma}\right] = 0$$

Isocurvature Fluctuations.



The total energy density is constant. Fluctuations in the radiation field are compensated by all other matter components.

$$\left.\begin{array}{l}\rho_m = m_X n_X \\ \rho_\gamma = \sigma T^4\end{array}\right\} \Rightarrow \quad \delta\rho = m_X \delta n_X + 4\sigma T^3 \delta T = 0 \quad \Rightarrow \quad \rho_X \frac{\delta n_X}{n_X} + 4\rho_\gamma \frac{\delta T}{T_o} = 0 \quad \Rightarrow$$

$$\frac{\delta T}{T_o} = -\frac{1}{4}\frac{\rho_X}{\rho_\gamma}\frac{\delta n_X}{n_X}$$

For modes outside the horizon, the behaviour is different in RD and in the MD regimes.

- In RD perturbations are isothermal: $\rho_X \ll \rho_\gamma \quad \Rightarrow \quad (\delta T/T_o) \sim 0$
- In MD large anisotropies will develop: $\rho_X \gg \rho_\gamma \quad \Rightarrow \quad (\delta T/T_o) \gg (\delta n_X/n_X)$

Inside the horizon, isocurvature perturbations evolve to become adiabatic.

Evolution of a Radiative Field in an Expanding Universe.

Let f be the distribution function of a gas of particles. If there are no particle creation, in an expanding Universe relativistic and non-relativistic particles verify $p \sim a^{-1}$, and

$$f(\vec{x}, \vec{p}) = \frac{d^6 N}{d^3 x d^3 p} \sim \frac{const}{a^3 a^{-3}} \sim const.$$

The distribution functions DO NOT GET DISTORDED with the expansion.

♠ For a planckian field:

$$f(\vec{x}, \vec{p}) = f(E/T)$$

Since with the expansion the energy of the photon distribution changes, so must do the temperature.

$$E \sim a^{-1} \qquad \Rightarrow \qquad T \sim a^{-1}$$

Dipole.

♣ The local motion of the observer produces a temperature anisotropy with a dipole pattern.

Let a radiative field be distributed uniformly on a volume for an observer O. For any observer O' that moves with velocity $\vec{v}$ with respect to O, the energy E of any photon will experience a boost:

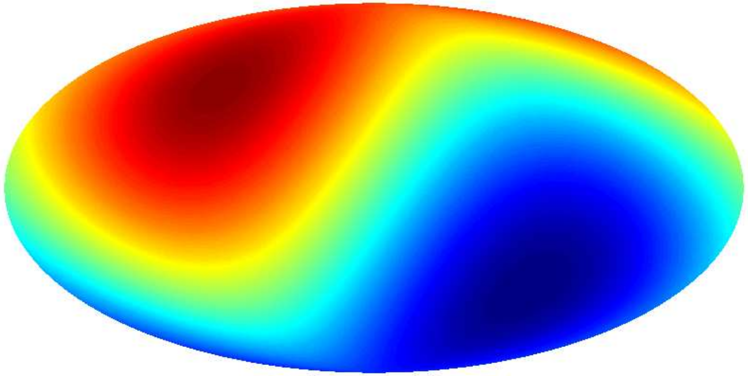
$$E' = E\gamma(1 - \vec{k}\vec{v}/c) = E\gamma(1 - \frac{v}{c}\cos\theta)$$

where θ is the angle between the photon and the direction of motion of the observer O'.

♠ Lorentz transformations do not affect the distribution function (Landau & Lifshitz, v.II), so the change boost in energy corresponds to a change in temperature:

$$T'(\theta) = T_o\gamma(1 - \frac{v}{c}\cos\theta) \quad \Rightarrow \quad \frac{\Delta T}{T_o} = \frac{T'(\theta) - T_o}{T_o} = \gamma(1 - \frac{v}{c}\cos\theta) \approx \frac{v}{c}\cos\theta$$

Earth motion gives rise to a dipole pattern.



Dipole Pattern

The Fixsen et al (1994, ApJ 420, 445) measurement of the CMB dipole was: 3.343 ± 0.016 mK (95% confidence level) with a direction $(\alpha, \delta) = (168^o.9, -7^o.5)$ that in Galactic coordinates are $(l, b) = (265^o.26, 48^o.74)$

Anisotropies due to the Gravitational Potential.

♣ The variation of the gravitational potential at the LSS induces temperature anisotropies (Sachs-Wolfe effect).

$$\frac{\Delta T(\vec{x_o}, \vec{n_e}, t_o)}{T_o} = \frac{1}{3}\Phi(\vec{x_e}, t_o)$$

The potentials will be evaluated today.

♣ Let us compute one observable of the temperature field: the correlation function.

$$\langle \Delta T(\vec{x}) \cdot \Delta T(\vec{x'}) \rangle = \frac{1}{9}\int \frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}\langle \phi(k)\phi^*(k')\rangle e^{i(\vec{k}\vec{x}-\vec{k'}\vec{x'})}$$

♣ Poisson's equation gives a relation between matter power spectrum and gravitational potential:

$$\nabla^2\phi(\vec{x}, t_o) = 4\pi G\bar{\rho}(t_o)\delta(\vec{x}, t_o)a^2(t_o) = \frac{3}{2}H_o^2\Omega_m\delta(\vec{x}, t_o) \quad \Rightarrow \quad \phi(\vec{k}, t_o) = \frac{3}{2}H_o^2\Omega_m\frac{\delta(\vec{k}, t_o)}{k^2}$$

After substituting the previous expression:

$$\langle \Delta T(\vec{x}) \cdot \Delta T(\vec{x'}) \rangle = \frac{1}{4} H_o^4 \Omega_m^2 \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \frac{\langle \delta(k)\delta^*(k') \rangle}{k^2 k'^2} e^{i(\vec{k}\vec{x} - \vec{k'}\vec{x'})}$$

Taking into account that $\langle \delta^*(\vec{k})\delta(\vec{k}') \rangle = (2\pi)^3 \delta^D(\vec{k} + \vec{k}')P(k)$ and the Rayleigh expansion of a plane wave:

$$e^{-i\vec{k}\hat{n}r} = 4\pi \sum_{lm} i^l j_l(kr) Y^*_{lm}(\Omega_{\hat{k}}) Y_{lm}(\Omega_{\hat{n}}),$$

we obtain

$$C(\theta) = \langle \Delta T(\vec{x}) \cdot \Delta T(\vec{x'}) \rangle = \frac{1}{4\pi} \sum (2l+1) C_l P_l(\cos\theta); \quad C_l = \frac{H_o^4}{2\pi} \int_o^\infty k^2 dk \frac{P(k)}{k^4} j_l^2(kR_H)$$

where θ is the angle between directions of observation $x\hat{n}$ and $x'\hat{n}'$ and $R_H = 2cH_o^{-1}$.

In the spetial case that the power spectrum behaves like a power law: $P(k) = Ak^n$ we get:

$$C_l = \frac{AH_o^{n+3}}{16} \frac{\Gamma(3-n)\Gamma\left(\frac{2l+n-1}{2}\right)}{\Gamma^2\left(\frac{4-n}{2}\right)\Gamma\left(\frac{2l+5-n}{2}\right)}$$

In the special case of $n = 1$ then

$$C_l = \frac{AH_o^4}{4\pi} \frac{1}{l(l+1)} = 6C_2 \frac{1}{l(l+1)}$$