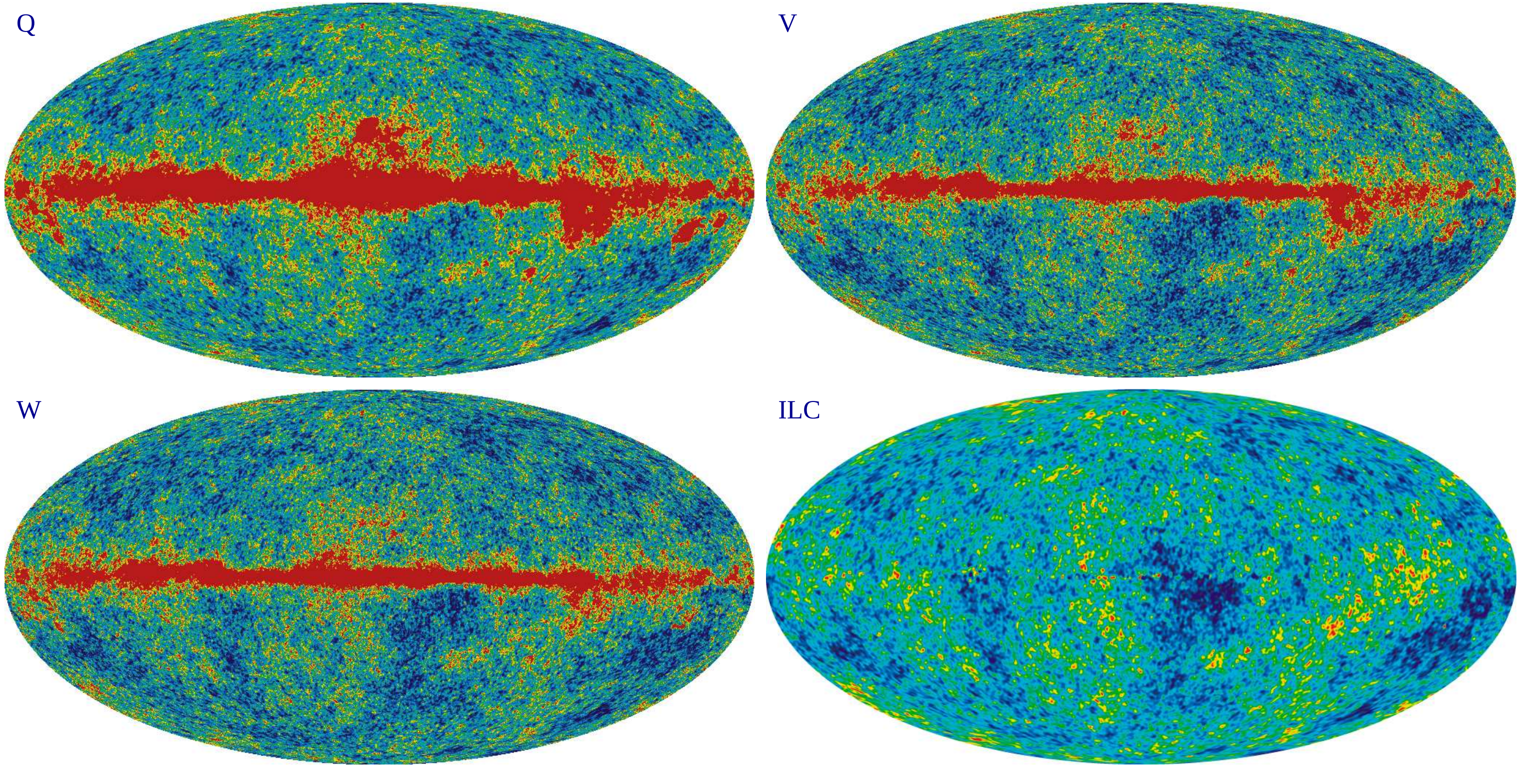
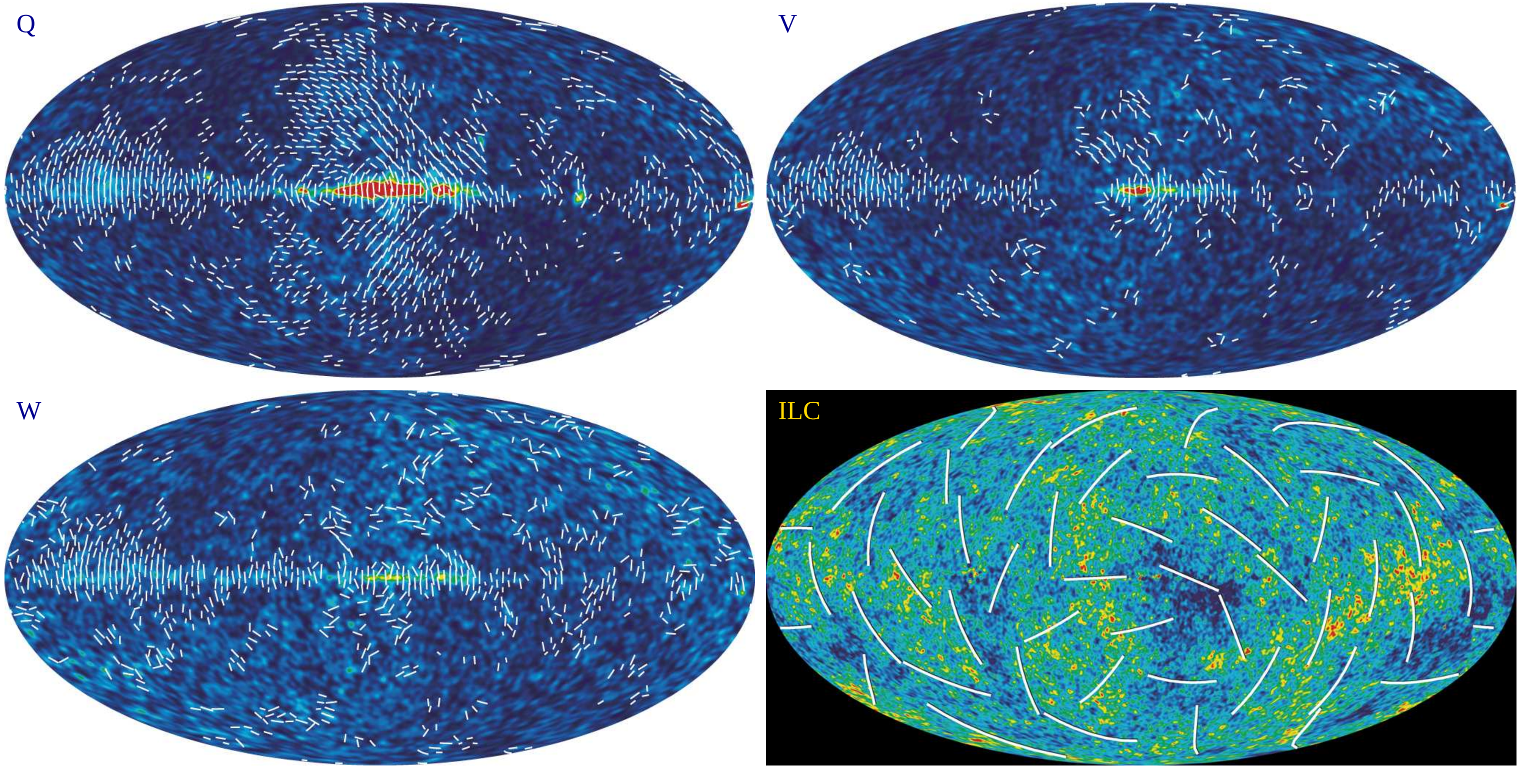
CMB OBSERVATIONS: WMAP 3yr DATA.

WMAP Temperature Data.



WMAP Polarization Data.



CMB Temperature Field.

♠ The basic observable of the CMB is its intensity as a function of frequency and direction on the sky $\hat{n}$.

♠ The CMB spectrum is an extemely good blackbody (Fixsen et al, ApJ 1996) with a nearly constant temperature across the sky.

♠ The temperature field is generally described in terms of its fluctuation:

$$\frac{\Delta T}{T_o}(\hat{n}) = \frac{T(\hat{n}) - T_o}{T_o} = \sum_{lm} a_{lm} Y_{lm}(\Omega_{\hat{n}}) \qquad \Longleftrightarrow \qquad a_{lm} = \int d\Omega_{\hat{n}} Y^*_{lm} \frac{\Delta T}{T_o}(\hat{n})$$

♠ If the temperature fluctuations are Gaussian, multipole moments are fully characterized by their power spectrum:

$$\langle a^*_{lm} a_{l'm'} \rangle = \delta_{ll'} \delta_{mm'} C_l$$

How accurately can the spectra be measured?.

♣ Models of structure formation predict the power spectrum of the radiation field, C_l. For each l we have $2l+1$ independent measurements. If the a_{lm}'s are Gaussian distributed, an optimal and unbiased estimate of the power at each l is:

$$C_l = \frac{1}{2l+1} \sum_{m=-l}^{m=+l} |a_{lm}|^2 \qquad \text{with rms} \qquad \Delta C_l = \sqrt{\frac{2}{2l+1}} C_l^{CMB}$$

The error introduced by having only $2l+1$ independent samples of power at each multipole moment is known as **Cosmic Variance**.

♣ If we average the power at different l in bands of width $\Delta l \approx l$, the error in the band-power is further reduced by a factor $l^{-1/2}$: the error on a band of width $(50-150)$ centered on $l=100$ is $\sim 1\%$.

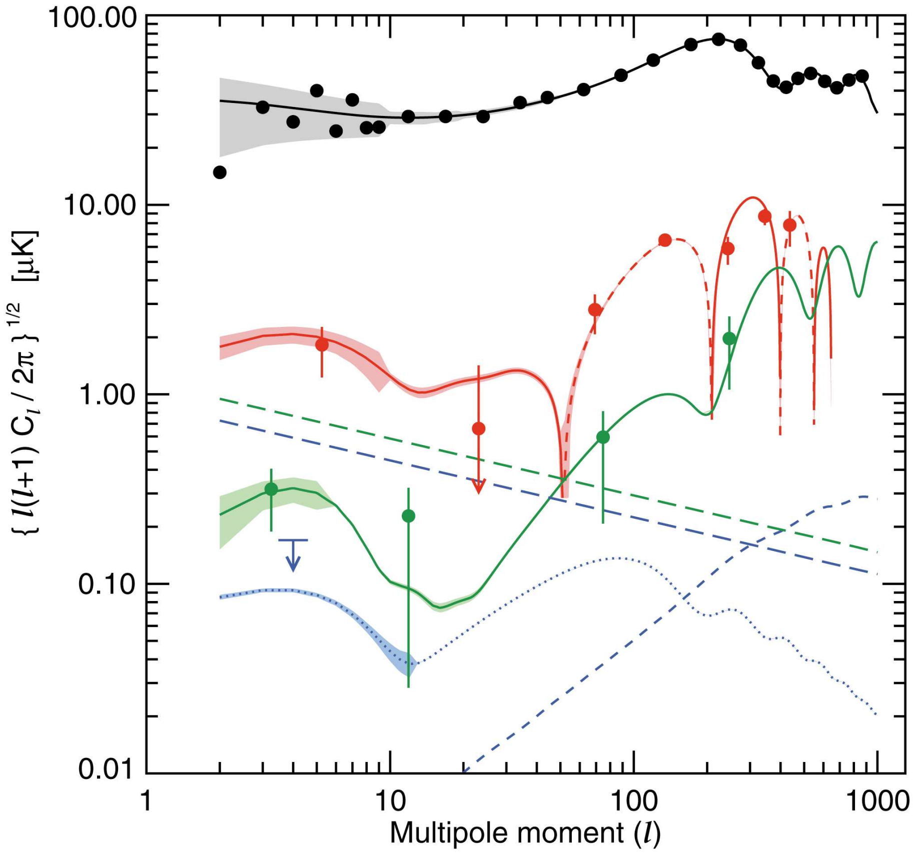
♣ Any source of noise, astrophysical or instrumental, increases the errors. If the noise is also Gaussian, with a known power spectrum then $\Delta C_l \sim C_l^{CMB} + C_l^N$.

♣ If the experiment does not sample the whole sky but only a fraction f_{sky}, then the errors increase by a fraction $f_{sky}^{-1/2}$. The resulting variance

$$\Delta C_l = \sqrt{\frac{2}{(2l+1) f_{sky}}} (C_l^{CMB} + C_l^N)$$

is known as **Sample Variance**.

Radiation Power Spectrum.



Recombination.

CMB and Big Bang Nucleosynthesis.

♠ To explain the observed He fraction is necessary that BBN be delayed and started with the production of D at $T \approx 0.1\mathrm{MeV}$. That temperature corresponds to a time scale: $t = [1\mathrm{MeV}/T]^2 \approx 100\mathrm{s}$.

♠ The cross section of strong interaction at that temperature is:

$$\sigma v(p + n \rightarrow D + \gamma) \simeq 5 \times 10^{-20}\mathrm{cm}^3/\mathrm{s}$$

♠ If the reaction rate is $\Gamma = t_{mfp}^{-1} = n_B \sigma v \sim 1$ then at that temperature: $n_B = 2 \times 10^{17}\mathrm{cm}^{-3}$

♠ The abundance of baryons today is $n(t_o) \sim a^{-3} \sim T_o^3$. Therefore:

$$n_B(t_o) T_o^{-3} = n_B(t_{BBN}) T_{BBN}^{-3} \qquad \Longrightarrow \qquad T_o = \left[\frac{n_B(t_{BBN})}{n_B(t_o)}\right]^{1/3} T_{BBN} \simeq 10\mathrm{K}$$

He synthesis in the early Universe requires the existence of a CMB of temperature $T \simeq 10\mathrm{K}$

Abundances of Different Particle Species in Thermal Equilibrium.

◇ Number density of relativistic particles

$$
\begin{aligned}
n &= \frac{\zeta(3)}{\pi^2} g T^3 \qquad (boson) \\
n &= \frac{3}{4}\frac{\zeta(3)}{\pi^2} g T^3 \qquad (fermion)
\end{aligned}
$$

◇ Number density of non-relativistic particles, valid for both bosons and fermions:

$$
n = g\left(\frac{mT}{2\pi}\right)^{3/2} \exp\left[-\frac{1}{T}(m-\mu)\right]
$$

Recombination.

◇ At $T \leq 13.6$eV neutral atoms may form. At that temperature, (p,e,H) are non-relativitic and their thermal abundances are given by:

$$n_i = g_i \left(\frac{m_i T}{2\pi} \right) \exp \left[\frac{\mu_i - m_i}{kT} \right]$$

where T is the equilibrium temperature common to all particles in the reaction and μ_i the chemical potential.

◇ The recombination reaction: $p + e \rightleftharpoons H + \gamma$ fixes $\mu_p + mu_e = \mu_H$ and yields:

$$n_H = \frac{g_H}{g_e g_p} n_p n_e \left(\frac{m_e T}{2\pi} \right)^{-3/4} \exp \left[\frac{B}{kT} \right]$$

where $B = m_p + m_e - m_H = 13.6$eV and $m_H \approx m_p$ except on the exponential.

If $x_e = n_e/n_B$ denotes the fraction of free electrons, then

$$\left.\begin{array}{l} n_H = n_B - n_p = \eta n_\gamma - n_e \\ g_\gamma = g_p = g_e = g_H/2 = 2 \\ n_\gamma = \frac{\zeta(3)}{\pi^2} g_\gamma T^3 \end{array}\right\} \Longrightarrow \frac{1-x_e}{x_e^2} = \sqrt{\frac{32}{\pi}}\zeta(3)\eta\left(\frac{T}{me}\right)^{3/2}\exp\left[\frac{B}{kT}\right]$$

Saha's Equation

We call RECOMBINATION the moment when 90% of the electrons are bound into neutral atoms.
If $x_e = 0.1$ then:

$$\eta = 2.68 \times 10^{-8}(\Omega_B h^2) \quad \Longrightarrow \quad 3.13 \times 10^{-18}(\Omega_B h^2)\left(\frac{1eV}{T}\right)^{3/2} = \exp\left[13.6\frac{1eV}{T}\right]$$

This equation must be solved by iteration.

A zero order solution is:

$$\frac{1eV}{T} = 3.084 - 0.0735\ln(\Omega_B h^2) \quad \Longrightarrow \quad \begin{cases} T \sim 0.3eV \ll 13.6eV \\ (1_z)_{rec} \approx 1367[1 - 0.024\ln(\Omega_B h^2)]^{-1} \end{cases}$$

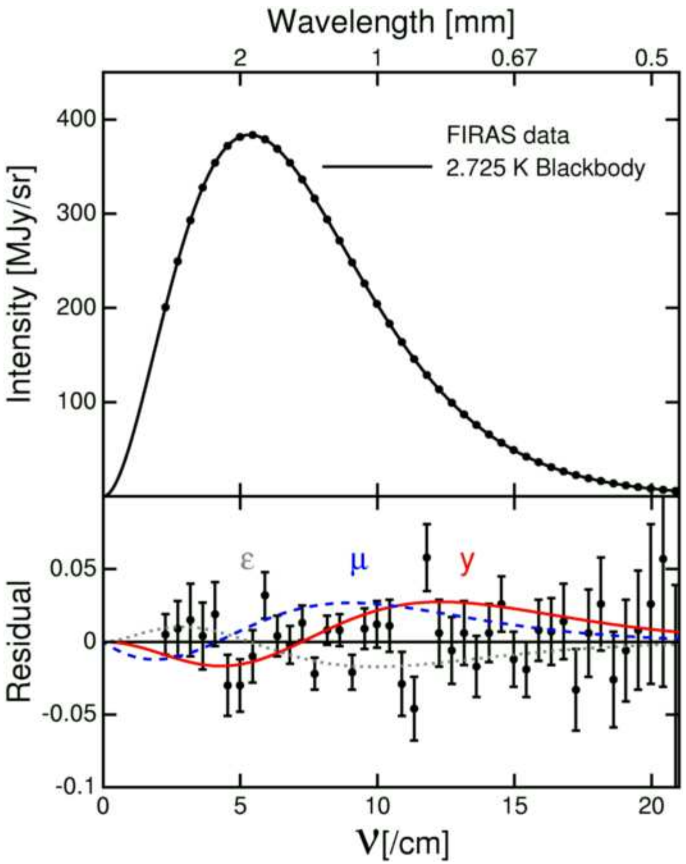
The very small baryon to photon ratio η delays the formation of neutral atoms from the expected temperature of 13.6eV to $T \sim 0.3$eV.

Decoupling.

Mean free path between photon electron collisions for different processes:

1. Thomson scattering; NO energy exchange and dominant at low redshifts:
   $t_{Th} = (c n_e \sigma_T)^{-1} = 6.14 \times 10^7 s (T/1eV)^{-3} (x_e \Omega_B h^2)^{-1}$
2. Compton scattering; Photon-electron energy exchange.
   $t_{Comp} = (n_e \sigma_T)^{-1} (mc/T) = 3.0 \times 10^{13} (T/1eV)^{-4} (x_e \Omega_B h^2)^{-1}$
3. Free-Free absorption:
   $t_{ff} = 1.1 \times 10^{11} s (\Omega_B h^2 x_e)^{-3/2} (T/1eV)^{-11/4}$
4. Double Compton scattering $(e + \gamma \rightarrow e + 2\gamma)$:
   $t_{2c} = 10^{20} s (T/1eV)^{-5} (\Omega_B h^2)^1$

Spectral Distortions.



Problems.

♠ (a) Consider an electron whichis moving with a velocity v through a radiation bath of temperature T. Show that the electron will feel a 'drag' force

$$F = -\frac{4\pi^2}{15}\sigma_T T^4 v$$

where σ_T is Thomson cross section. (b) Assume now that a bath of electrons of temperature T_e crosses the bath of photons. Show that the net rate of transfer of energy from radiation to matter per electron is

$$\frac{dQ}{dt} = \frac{4\pi^2}{15}\sigma_T \left(\frac{T^4}{m}\right)(T - T_e)$$

(c) It the matter is fully ionized, then the kinetic energy per electron is $3T_e$. Therefore, using the previous expression, show that the change in temperature is

$$\dot{T}_e = \frac{4\pi^2}{45}\sigma_T\left(\frac{T^4}{m}\right)(T - T_e)$$

How would this equation be modified in a expanding Universe, with only a fraction of the matter ionized.