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MINIMAL COSMIC BACKGROUND FLUCTUATIONS IMPLIED BY STREAMING MOTIONS

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ABSTRACT

Using a new, power spectrum–independent approach, we derive the minimal cosmic background radiation anisotropy implied by the presence of large-scale streaming motions. If the tentative evidence for deviations from the Hubble flow of magnitude $\delta V/V \approx 0.1$ at $V \approx 5000$ km s^{-1} is confirmed, we predict microwave background fluctuations with a coherence scale $\sim 2°$ and dispersion $\delta T/T > 10^{-5}$. If the observational limits on $\delta T/T$ were to be reduced below our minimal predictions, then gravitational instability without reheating as a mechanism for generation of the large-scale structure of the universe would be in severe difficulty.

Subject headings: cosmic background radiation — cosmology

I. INTRODUCTION

Several groups have recently presented observational results confirming a decade-old claim, known as the Rubin-Ford effect (Rubin *et al.* 1976), that the Hubble expansion on scales $r \approx 50h^{-1}$ Mpc is perturbed by bulk velocities $v \approx 600$ km s^{-1} (Collins, Joseph, and Robertson 1986; Burstein *et al.* 1986; Dressler *et al.* 1987; James, Joseph, and Collins 1987; Aaronson *et al.* 1988). Here h is the Hubble constant, H_0, in units of 100 km s^{-1} Mpc^{-1}. Such measurements can be used to test models of the origin of structure in the universe (Clutton-Brock and Peebles 1981; Kaiser 1983; Vittorio and Silk 1985). Indeed, confirmation of the Rubin-Ford effect would challenge the cold dark matter (CDM) scenario for the formation of large-scale structure (Vittorio, Juszkiewicz, and Davis 1986; Bond 1986), as well as the cosmic string model (Bertschinger 1987; Melott and Scherrer 1987; Shellard *et al.* 1987; van Dalen and Schramm 1987). To explain these observations, new scenarios with enhanced large-scale fluctuations have been constructed. One possibility is an open baryon-dominated model (Peebles 1987), which has a power spectrum with a prominent feature near the matter-radiation Jeans wavelength $\lambda_J = 50(\Omega h^2)^{-1}$ Mpc (where Ω is the cosmological density parameter at present time). Other options, including nonstandard inflationary scenarios (Vittorio, Matarrese, and Lucchin 1987), an ad hoc modification of the $\Omega = 1$ CDM model (Bardeen, Bond, and Efstathiou 1987), and open CDM + baryon universes (Blumenthal, Dekel, and Primack 1987) have also been considered.

Despite the reported agreement between various recent observations and the original Rubin-Ford result, the interpretation of these difficult measurements still remains uncertain. The existing estimates of the amplitude of the velocity derived from different data sets range from ~ 200 km s^{-1} (Aaronson *et al.* 1986) to ~ 1000 km s^{-1} (Collins, Joseph, and Robertson 1986). The estimates of the scale of the mean drift for the sample of elliptical galaxies range from $60h^{-1}$ Mpc (Dressler *et al.* 1987), to $15h^{-1}$ Mpc (Frenk and Lucey 1987; N. Kaiser, private communication).

These uncertainties have motivated us to look for alternative ways of verifying the Rubin-Ford effect and investigate its observable consequences, other than the motions themselves. A simple order-of-magnitude estimate suggests that the cosmic background radiation (CBR) anisotropy implied by the existence of streaming motions might be observable with present technology in the near future. In an $\Omega = 1$ universe, a small velocity perturbation v on a scale r implies the existence of a mass inhomogeneity with a fractional density contrast $\delta \approx v/(H_0 r)$. The peculiar gravity of such an inhomogeneity induces a perturbation of the CBR temperature $\delta T/T \approx \delta(r/R)^2$ on the last scattering surface, where $R = c/H_0$ is the Hubble radius now (Peebles 1980). In terms of the velocity perturbation, this amounts to

$$\delta T/T \approx (vr/cR) \approx 3 \times 10^{-5} v_{500} r_{50}\,, \qquad (1)$$

where $v_{500} = v/(500 \text{ km s}^{-1})$ and $r_{50} = rh/(50 \text{ Mpc})$. The anisotropy appears on an angular scale $\Theta \approx (r/R) \approx 1° r_{50}$. The above estimate does not apply to models where the velocity field is driven by nongravitational forces such as explosions caused by supernovae (Ostriker and Cowie 1981) or superconducting cosmic strings (Ostriker, Thompson, and Witten 1986). However, it is not clear if such models can provide enough energy to generate coherent flows on length scales discussed here (Peebles 1988).

Our aim in this *Letter* is to calculate the minimal CBR anisotropy implied by the presence of peculiar motions of a given amplitude on some specified scale. We attempt to make this calculation as model-independent as possible. We assume that (1) peculiar velocities are driven purely by the gravity of random, Gaussian-distributed density fluctuations and that (2) there was not secondary ionization of the intergalactic medium. We *do not* assume any specific spectrum of primeval

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density perturbations. Moreover, our calculation does not depend on the occurrence of "biasing," as both the peculiar velocity and radiation temperature fields respond to the mass inhomogeneities. (The "biasing" hypothesis implies that bright galaxies are more clustered than the mass, and the model predictions for $\delta T/T$ depend on the assumed "biasing factor" when counts of luminous galaxies are used to normalize the spectrum, see, e.g., Bond 1987).

II. THE RADIATION BRIGHTNESS AND THE VELOCITY FIELD

The existence of streaming motions of magnitude v on scale r constrains the power spectrum of the density fluctuations, $P(k)$, where k is the wavenumber. If v denotes the rms value of the peculiar velocity expected on scale r, then using the linearized equation of continuity (Peebles 1980) and a Gaussian model for the observational selection function, we obtain

$$v^2(r) = \frac{\Omega^{1.2}H_0^2}{2\pi^2}\int_0^\infty P(k)\,e^{-k^2r^2}\,dk. \tag{2}$$

Since the observational status of v (and the corresponding r) is rather uncertain, we will keep v and r as free parameters in all expressions.

The gravitational potential fluctuations that perturb the Hubble flow will also induce anisotropy in the cosmic microwave background radiation. The response of photons to mass density gradients in an $\Omega = 1$ universe was calculated by Sachs and Wolfe (1967). For temperature fluctuations across angles $\Theta \ll \Omega$ radians, their result can readily be generalized to models with $\Omega < 1$ (Peebles 1981). In this approximation, the autocorrelation function for fractional radiation temperature fluctuations Δ is given by

$$C(\Theta) = \langle\Delta(0)\,\Delta(\Theta)\rangle = \frac{F^2(\Omega)}{8\pi^2R^4}\int_0^\infty \frac{P(k)}{k^2} j_0(2k\Theta R/\Omega)\,dk, \tag{3}$$

where $\langle\cdots\rangle$ denotes ensemble averaging, $F(\Omega) = 2(1-\Omega)/5D_1(\Omega)$, and $j_0(y) = \sin(y)/y$. The exact expression for $D_1(\Omega)$ is given by Peebles (1980, eq. [11.16]). Here we use an approximate formula $F(\Omega) = \Omega^{0.3}$. Expression (3) is valid provided $30h^{-1}$ arcmin $\lesssim \Theta \ll \Omega$ rad (see § II*c* below). It also assumes a perfectly narrow beam antenna. For a receiver with a Gaussian beam response of dispersion $\theta/2$, corresponding to full width at half-maximum (FWHM) of 1.2θ, the correlation function is $C(\Theta,\theta) = 2\int_0^\infty C(\Phi)\Phi\exp[-(\Theta^2+\Phi^2)/\theta^2]I_0(2\Theta\Phi/\theta^2)\,d\Phi/\theta^2$ (Wilson and Silk 1981), where I_0 is a modified Bessel function and $C(\Phi) \equiv C(\Phi,0)$ is the intrinsic correlation function, described by equation (3). The rms temperature difference, expected in a beam-switching experiment with beamthrow angle Θ and a beamwidth θ, is

$$\delta T/T = \sqrt{2C(0,\theta) - 2C(\Theta,\theta)}\,. \tag{4}$$

a) *The Minimal* $\delta T/T$

Let us find the power spectrum that minimizes the variance of the temperature fluctuations, $C(0)$, subject to the condition that the rms velocity at a scale r is fixed. This problem is equivalent to minimizing the functional

$$\phi\{P(k)\} = \frac{\int_0^\infty P(k)\,k^{-2}\,dk}{\int_0^\infty P(k)\exp(-k^2r^2)\,dk}. \tag{5}$$

Now, let us rewrite the integrand in the numerator as $f(k)P(k)\exp(-k^2r^2)$. The function $f(k) = k^{-2}\exp(k^2r^2)$ has a global minimum at $k = 1/r$. Hence, $f(k) \geq er^2$ and as $P(k) \geq 0$, the numerator in equation (5) is always $\geq er^2 \times$ denominator. This implies $\phi\{P(k)\} \geq er^2$ for any $P(k)$. Clearly, $P(k) \propto \delta_D(k - 1/r)$, where δ_D is the Dirac delta function, is a solution, as for this choice ϕ reaches its minimal value. Using equation (2) to fix the normalization, we finally obtain

$$P(k) = (2\pi^2v^2e/H_0^2\Omega^{1.2})\,\delta_D(k - 1/r). \tag{6}$$

By definition, $P(k)$ is always positive definite. This guarantees that there are no minimal solutions other than the above.[3] The angle subtended by the comoving scale r on the last scattering surface is $r\Omega/2Rr$. For a beam-switching experiment with $\theta \ll r\Omega/2R \ll \Theta$, we have $(\delta T/T)^2 = 2C(0,0)$. The minimal rms value of the temperature anisotropy is then

$$\delta T/T = (vr/Rc)\,\Omega^{-0.3}\sqrt{e/2}, \quad \text{for} \quad \Theta \gg r\Omega/2R \gg \theta. \tag{7}$$

For $\Omega = 1$, the above result differs from the earlier naive estimate (eq. [1]) only by a factor $\sqrt{e/2} = 1.17$. The relation (7), together with the measurements of v and r, gives a lower bound on the expected $\delta T/T$ and can be used to test the consistency of large-scale streaming motions with gravitational instability models for the growth of structure.

It is straightforward to invert the above argument by solving for a $P(k)$ that maximizes the rms velocity on a fixed scale r: $v(r) =$ max under the condition that the rms intrinsic CBR anisotropy $\delta T/T$ is given. It turns out that the power spectrum that solves this problem is also proportional to $\delta_D(k - 1/r)$, while its normalization is now fixed by $\delta T/T$. As a consequence, the resulting relation between $\delta T/T$, v, and r is identical to equation (7). Hence, a detection or an upper limit on the CBR anisotropy can be used to set an upper limit on the amplitude of the large-scale peculiar velocity.

[3] Indeed, for any solution $P(k)$, $\phi(P) - er^2 = 0$. We first introduce $g(k) = k^{-2} - r^2\exp(-k^2r^2+1)$. Then we can rewrite this as $\int_0^\infty P(k)g(k)\,dk = 0$. As $g(k) > 0$ for all k except $k = 1/r$ and $P \geq 0$ always, $P(k)$ has to vanish everywhere except $k = 1/r$. This excludes all solutions except the power spectrum, given by eq. (6).

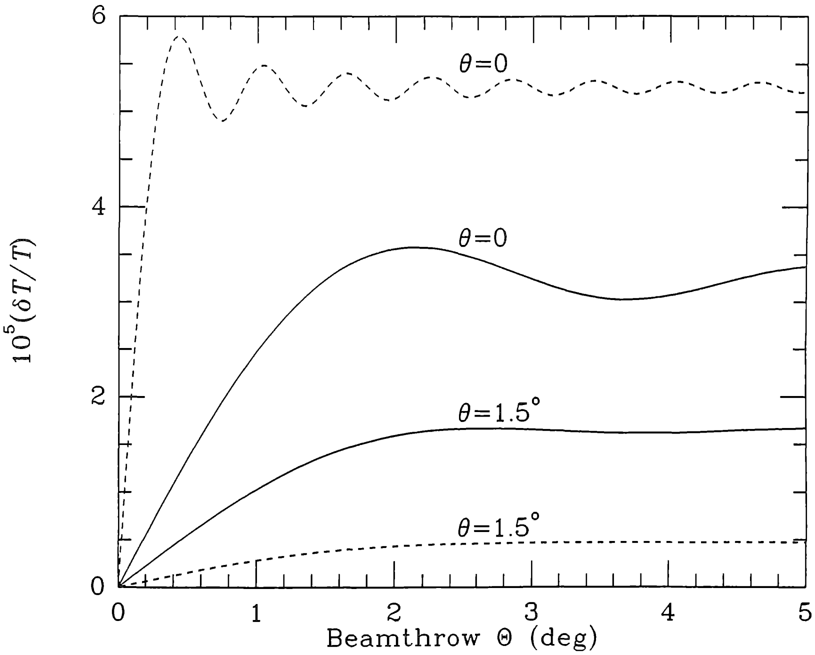

FIG. 1.—Root mean square temperature differences are shown for two minimal models with $v_{500} = r_{50} = 1$, $\Omega = 1$ (*solid line*) and $\Omega = 0.2$ (*dashed line*), respectively, each plotted for two antenna beamwidths θ. The intrinsic anisotropy ($\theta = 0$) is enhanced when $\Omega < 1$ due to the slower growth of inhomogeneities. On the other hand, the focusing of geodesics reduces the coherence angle $\Theta_* = 2.15°\Omega r_{50}$ of the temperature fluctuations and the signal is strongly attenuated in receivers with beamwidths $\theta > \Theta_*$.

b) *Can We "See" Primordial Potential Wells?*

The intrinsic temperature autocorrelation function for our minimal model is

$$C(\Theta) = e(vr/2cR)^2\Omega^{-0.6}j_0(2R\Theta/r\Omega), \tag{8}$$

and $\delta T/T$ peaks at the separation

$$\Theta_* = 2.25\Omega(r/R) = 2°.15\Omega r_{50}, \tag{9}$$

corresponding to the first maximum of the function $1 - j_0(2\Theta R/r\Omega)$. Using equation (8) with $r_{50} = v_{500} = 1$ and equation (4), we have calculated $\delta T/T$ numerically for two beamwidths θ and two values of Ω. The results are plotted in Figure 1.

The mean numbers and angular sizes of regions at various levels of brightness expected to appear in the sky are essentially governed by the ratio of two parameters (Sazhin 1985; Zabotin and Nasel'skii 1985): $\sigma^2(\theta) \equiv C(0, \theta)$ and $u^2(\theta) \equiv -d^2C(\Theta, \theta)/d\Theta^2$, evaluated at $\Theta = 0$. The number of high peaks ("hot spots") of height $\Delta \geq \nu\sigma$ or of "cold spots" with $\Delta \leq -\nu\sigma$, where $\nu \geq 2$, is approximately given by $N(\nu) = \sqrt{2/\pi}\,(u/\sigma)^2\nu \exp(-\nu^2/2)$. The mean angular diameter of the isotherms $\Delta = +\nu\sigma$ is $D(\nu) = 2\sqrt{2}\,\sigma/\nu u$, while the mean surface area of the sky containing either a hot or a cold spot with $|\Delta| = \nu\sigma$ is given by $S(\nu) = -2\pi(dN/d\nu)^{-1}$ (Bond and Efstathiou 1987; Vittorio and Juszkiewicz 1987). For the minimal $C(\Theta)$ (eq. [8]), convolved with a Gaussian antenna beam, $D(\nu) = 1.1q\Theta_*/\nu$ and $S(\nu) = 5.4(\nu^2 - 1)^{-1} \exp(\nu^2/2)(\Omega q r_{50})^2$ deg^2. Here the parameter $q(\theta) \equiv 2.6\sigma/\nu\Theta_*$ describes the difference between the *intrinsic* sky pattern of the CBR and the *apparent* pattern, expected to be observed, using a detector with a specific beamwidth θ. We have defined q so, that for a perfectly narrow beam antenna ($\theta = 0$), $q = 1$. For finite beamwidths, $q(\theta)$ is well approximated by $q \approx \exp(t^2/2)$ for $t \lesssim 1$, and $q \approx 2t - 0.35$ for $t \gtrsim 1$, with $t \equiv \theta/\Theta_*$. A map of the sky made using an antenna of small beamwidth $\theta \ll \Theta_*$ would reveal the true structure of primordial potential wells with $D(\nu) \approx \Theta_*/\nu$. An antenna with $\theta \gg \Theta_*$ would give a resolution-limited image of the sky with a coherence scale defined by θ. In this regime $D(\nu) \approx 2\theta/\nu$ and the number of hot spots is reduced by the factor q^2.

The formulae for $S(\nu)$ and $D(\nu)$ derived here can help plan future anisotropy measurements. For example, for a beamwidth $\theta = 30''$, $\Omega = 1$, and $v_{500} = r_{50} = 1$, $S(2) = 13.7$ deg^2. Hence, it should suffice to scan a field of 4° by 4° to discover a hot or cold spot with amplitude $\Delta = 2\sigma \approx \pm 4 \times 10^{-5}$ and a diameter $D(2) \approx 1°$.

c) *The Acceptable Range of Model Parameters*

We have hitherto treated the velocity coherence length r as a free parameter of arbitrary value. However, there is a lower limit for acceptable values of r. We assume that the Sachs-Wolfe effect dominates over the Doppler and adiabatic contributions to $\delta T/T$ and ignore the effects of radiation pressure and Thomson scattering. The effect of the fuzziness of the last scattering surface can be modeled by replacing $C(0)$ with $C_{\rm LS}(0) \propto \int_0^\infty P(k)k^{-2} \exp(-k^2 r_{\rm LS}^2)\, dk$, where $r_{\rm LS} \approx 5(\Omega h^2)^{-1/2}$ Mpc (Sunyaev 1978; Jones and Wyse 1985;

Bond 1987). The corrected minimal solution is $P(k) \propto \delta_D[k - (r^2 - r_{LS}^2)^{-1/2}]$; for $r \le r_{LS}$ the minimal solution does not exist. This is not surprising since on sufficiently small scales Thomson scattering damps $\delta T/T$ and $v(r)$ can be considerable without leaving an imprint in the microwave sky. Hence, our calculation requires $r \gg r_{LS}$. The effects of pressure can be neglected for Fourier components with wavelengths exceeding the matter-radiation Jeans length, $\lambda_J = 50(\Omega h^2)^{-1}$ Mpc (Peebles 1980). A perturbation of comoving length λ_J, straddling the last scattering surface, subtends an angle $\sim 30h^{-1}$ arcmin. The minimal $P(k)$ contains only one spectral line with a wavelength $2\pi/k = 2\pi r$. Since $(\lambda_J/2\pi) > r_{LS}$, both pressure and scattering can be safely ignored if $r \gg \lambda_J/2\pi$ and $\Theta > 30h^{-1}$arcmin.

III. IMPLICATIONS

One of the recent experiments (Davies *et al.* 1987) used three beams with $\theta = 7\overset{\circ}{.}1$ and total antenna response consisting of a central positive beam with a negative beam of half the amplitude displaced by an angle $\Theta = 8\overset{\circ}{.}2$ on either side. The predicted variance of these measurements is $(\Delta T/T)^2_{DL} = 1.5C(0, \theta) - 2C(\Theta, \theta) + 0.5C(2\Theta, \theta)$. If $\Omega = 1$ and the velocity at $r_{50} = 1$ is v_{500}, we expect

$$(\Delta T/T)_{DL} \ge 2 \times 10^{-6} v_{500} . \tag{10}$$

In fact, Davies *et al.* reported a detection at the level of 5×10^{-5}, and Lasenby (1988) reported that this was unlikely to be affected by any galactic emission.

P. Timbie and D. Wilkinson (talk given at the "ΔT over Tea" Workshop, Toronto, 1987) used a beam pattern $W(x, y) \propto \exp[-2(x^2/\theta_1^2 + y^2/\theta_2^2)] \cos(\kappa x)$, where $\kappa = 70.4$, $\theta_1 = 4\overset{\circ}{.}41$, $\theta_2 = 4\overset{\circ}{.}58$ and $|x| \le 4°$, $|y| \le 5°$ are Cartesian coordinates for the angular position in the sky. Outside of the 8° by 10° rectangle $W(x, y) = 0$. This beam was wobbled between two positions separated by an angle $\Theta = 8°$ along the x-direction. The predicted variance of these measurements is $(\Delta T/T)^2_{TW} = 2C_{TW}(0) - 2C_{TW}(\Theta)$, where C_{TW} is the intrinsic $C(\Theta)$, convolved with the beam. If $\Omega = 1$ and the velocity at $50h^{-1}$ Mpc is v_{500}, we expect

$$(\Delta T/T)_{TW} \ge 1.4 \times 10^{-5} v_{500} , \tag{11}$$

or with $v_{500} \approx 1$, a value slightly below the sensitivity of this experiment (P. Timbie, private communication).

An experiment now under preparation at Lawrence Berkeley Laboratories (P. Lubin, private communication) will use two beams with FWHM = 20′, separated by an angle $\Theta \approx 1°$. Using equations (4) and (8), we predict, for $\Omega = 1$ and velocity v_{500} at $r_{50} = 1$,

$$(\Delta T/T)_L \ge 2.4 \times 10^{-5} v_{500} , \tag{12}$$

which is several times larger than the expected sensitivity of this experiment if $v_{500} \approx 1$. At 95% confidence, each of these three lower limits (10)–(12) should be increased by a factor of 2.

We have calculated the predicted rms signal in the three experiments described above for a family of $\Omega = 1$ minimal models, parameterized by the scale of the coherent motion r and the rms value of the velocity, $v(r)$. The results are plotted in Figure 2. Using equation (4), we have also computed the minimal rms anisotropy expected in a beam-switching experiment with large Θ: $\delta T/T = \sqrt{2C(0, \theta)}$. The results for two minimal models are presented in Table 1. We consider $\Omega = 1$ and 0.2. Each model is normalized to yield $v(r) = 500$ km s^{-1} at $r_{50} = 1$. For comparison, we present predictions of

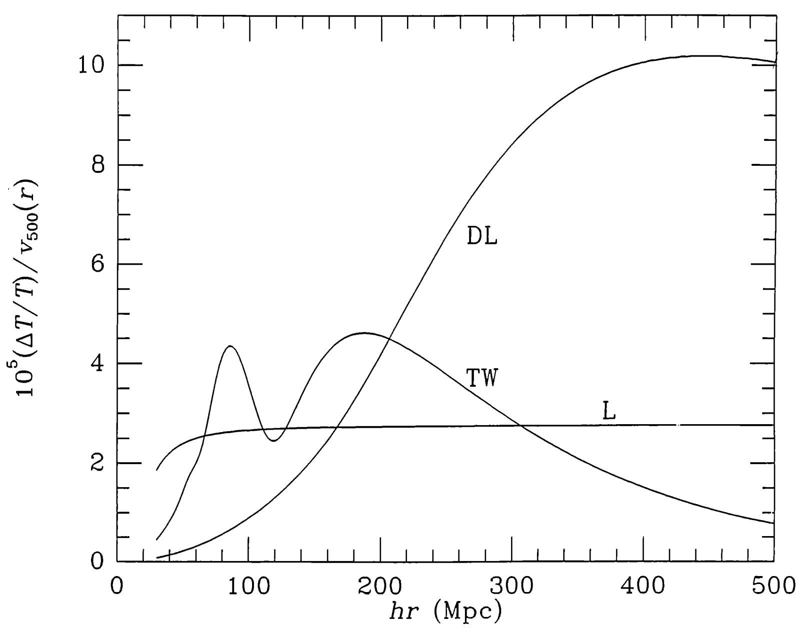

FIG. 2.—Root mean square temperature differences are given for a family of $\Omega = 1$ minimal models, parameterized by the spatial scale of the flow r and the rms value of the velocity, $v = 500 v_{500}$ km s^{-1}. The three curves correspond to different experimental configurations discussed in the text: DL, Davies *et al.* (1987); TW, P. Timbie and D. Wilkinson (1987, in preparation); L, future LBL experiment (P. Lubin, private communication).

TABLE 1

MINIMAL VERSUS "STANDARD" PREDICTIONS FOR $\delta T/T$

Model[a]	(Ω, h)	$v(50)$[b]	$10^5\delta T/T$: $\theta^c = 0$	$\theta = 2.5$	$\theta = 6$
HDM	(1, 0.5)	423	5.87	4.67	4.08
CDM	(1, 0.5)	149	2.13	1.74	1.53
CDM + mountain	(1, 0.5)	500	5.89	4.59	2.67
Minimal	(1.0, h)	500	3.24	0.92	0.37
Minimal	(0.2, h)	500	5.25	0.28	0.12

[a] HDM and CDM models are normalized in a standard way, assuming that light traces mass (see text). The remaining models are normalized to $v(r) = 500$ km s^{-1} at $r = 50h^{-1}$ Mpc. For minimal models h is arbitrary.

[b] Rms peculiar velocity at $50h^{-1}$ Mpc in units of km s^{-1}.

[c] Unit = degrees.

a "CDM + mountain" model à la Bardeen, Bond, and Efstathiou (1987), normalized in the same way. We also include a hot dark matter (HDM) model normalized so that the first nonlinear structures appear at a redshift $z = 3$, and a CDM model normalized to unit mass variance within spheres of radius $= 8h^{-1}$ Mpc (no "biasing"). For a more detailed description of the normalization adopted here, see Vittorio and Silk (1985). The HDM and "CDM + mountain" models that can account for large scale drift velocities yield $\delta T/T$ in excess of the predictions of our minimal model, as they must. The standard CDM model at $r = 50h^{-1}$ Mpc predicts $v \ll 500$ km s^{-1}, and consequently the CBR anisotropy in this scenario is smaller than in the minimal model. Note that the minimal anisotropy predictions do not depend on H_0, are only weakly dependent on Ω, and fall off rapidly with antenna beamwidth, in contrast with various "conventional" models. The reduction in $\delta T/T$ for $\theta \gg 1°$ is caused by averaging over many potential wells and barriers (or in terms of sky brightness, hot and cold spots); hence the difference between the predictions for various experimental configurations (eqs. [10]–[12]) as seen both in Figure 1 and in Table 1.

As we have shown (eq. [7]), one can also set an upper bound on $vr\Omega^{-0.3}$, given an upper limit or detection of $\delta T/T$. A limit $\delta T/T \le 3 \times 10^{-5}$ established for $\Theta \gg \Theta_* \gg \theta$ (a result which should soon become available) would imply an important constraint on the amplitude of the large-scale velocities $v < 500\Omega^{0.3}r_{50}^{-1}$ km s^{-1}.

Our goal here was to estimate the minimal CBR anisotropy implied by the large-scale streaming motions. We have found that the existing upper limits on $\delta T/T$ are not inconsistent with $v(r) = 500$ km s^{-1} at $r = 50h^{-1}$ Mpc. A reduction of the observational limits on the CBR anisotropy below our minimal predictions for $\delta T/T$ would challenge the current interpretation of measurements of deviations from the Hubble flow. If, however, the measurements of large-scale motions acquire a firm status and no associated CBR anisotropy on the appropriate angular scale of $\Theta \approx 2°\Omega r_{50}$ is established by future experiments, then gravitational instability as a mechanism for generation of the large-scale structure of the Universe would be in severe difficulty unless one appeals to early reheating of the intergalactic medium.

Finally we remark that our approach, based on spectrum-independent integral inequalities can be also used to constrain other measures of the large-scale structure, e.g., the variance of mass fluctuations (Suto *et al.* 1987).

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